Corner Detection & Optical Flow

CSC420
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Slide credit: Babak Taati ←Ahmed Ashraf ←Sanja Fidler

Logistics

- A2 due on Friday

Overview

- Recap
- Image features
- Corner detection
- Optical flow

Recap

Review

- Images
  - composed of individual pixels
- Filtering
  - extracting structure from a collection of pixels
- Convolution
  - mathematical operation that performs filtering
  - "convolution theorem"
- Smoothening
  - e.g., via Gaussian filter
- Edges
  - simplified representation of images
  - how related to image derivatives?
- Image resizing
  - what is an image pyramid?
  - what is aliasing?
  - how can we upsample an image?
  - what is an upsampling filter?

Image Features:
Interest Point (Keypoint) Detection

Image Features

- What skyline is this?

Image Features

- What skyline is this?

Image Features

- What skyline is this?

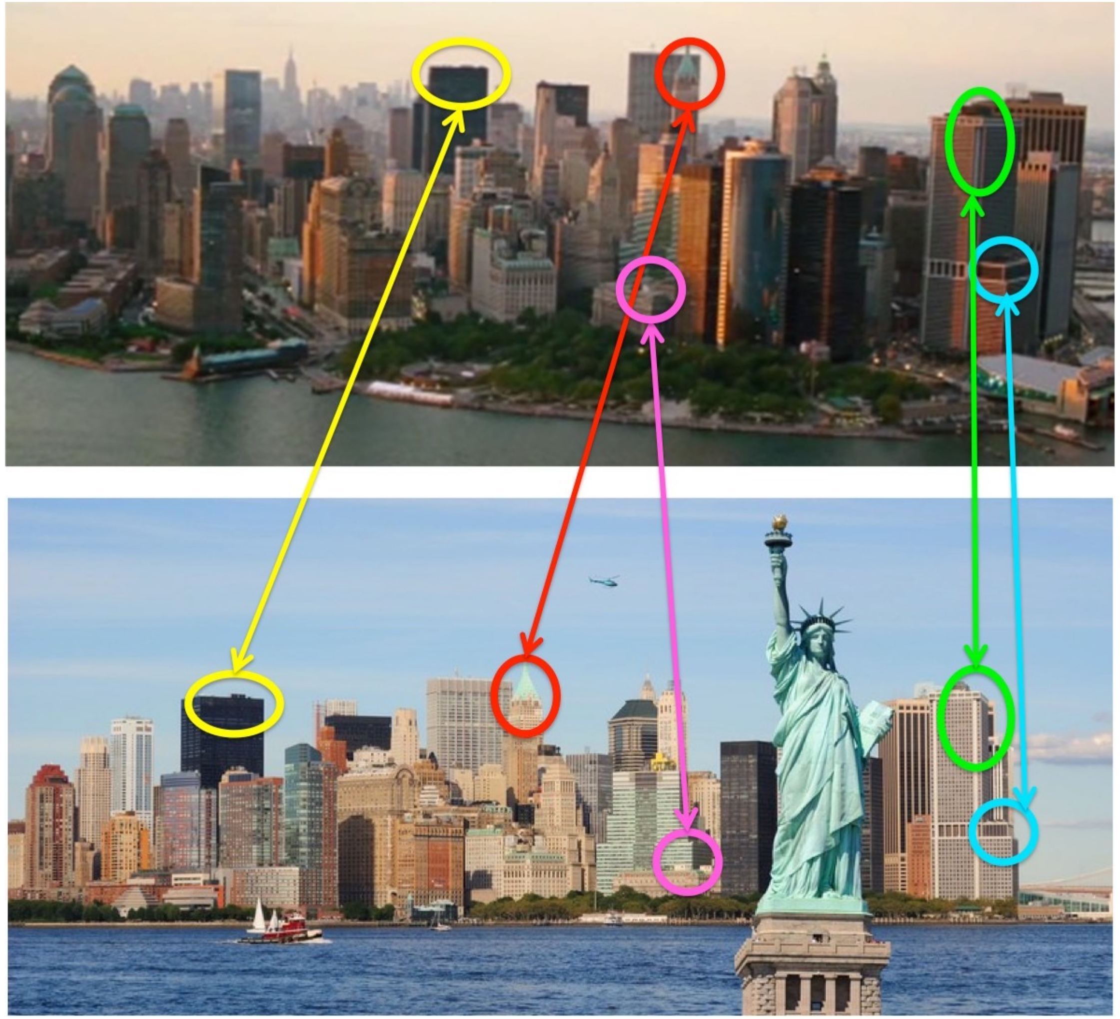
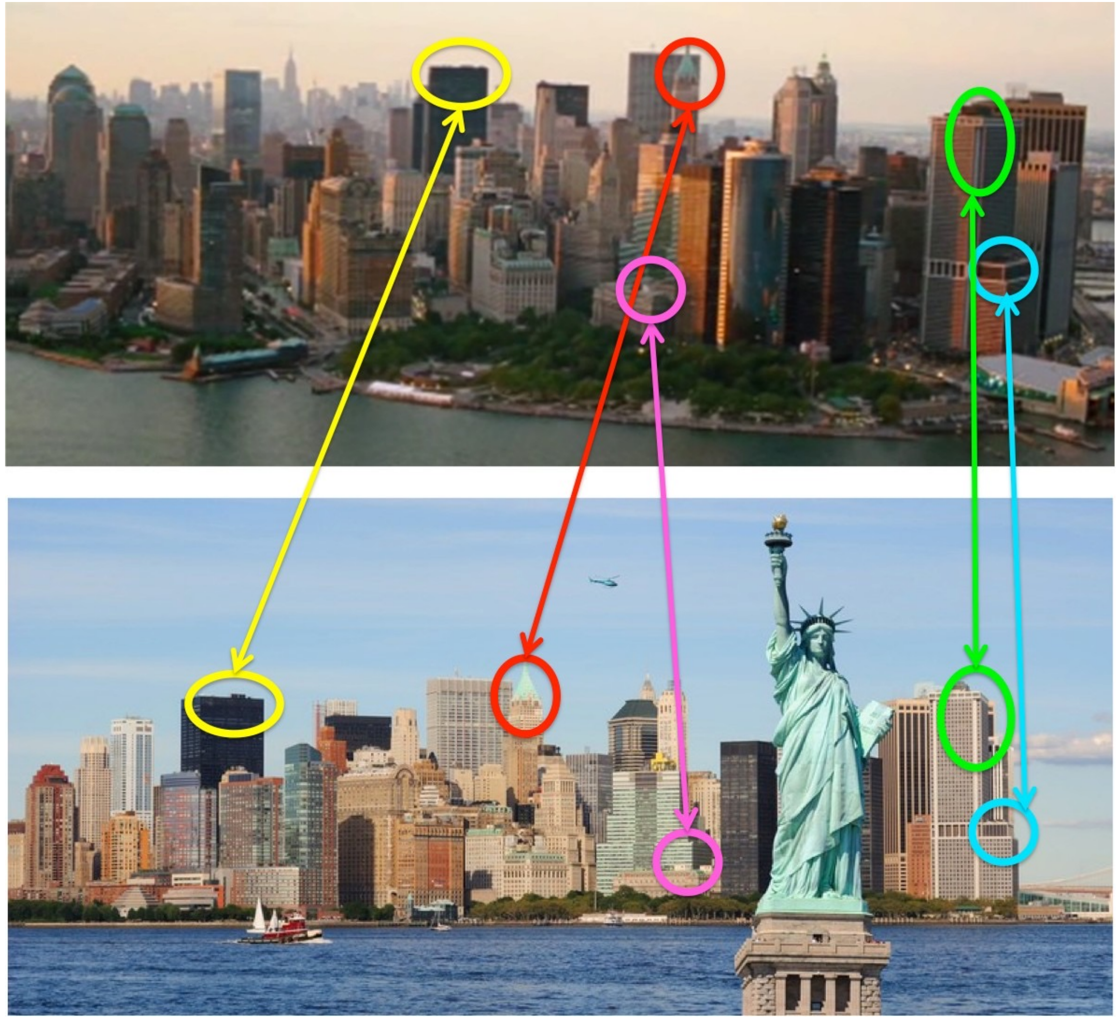


Image Features

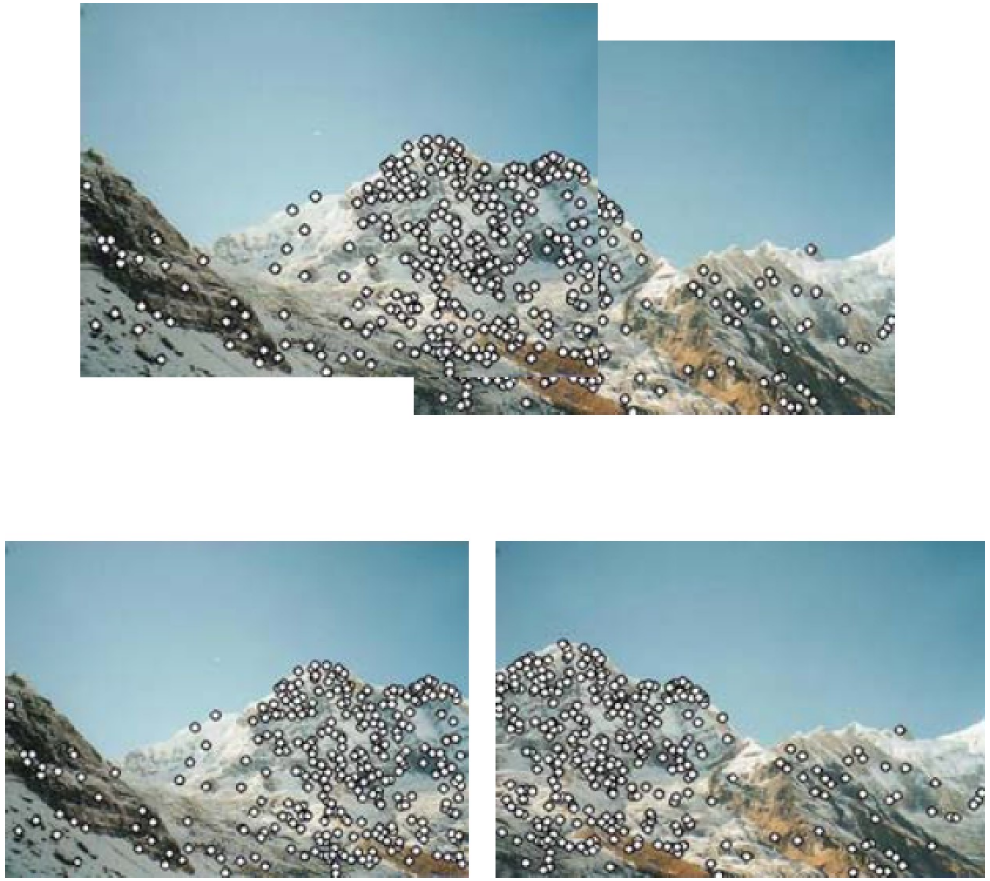
- What skyline is this?

We matched in:

- Distinctive locations: keypoints
- Distinctive features: descriptors



Application Example: Image Stitching



[Source: K. Grauman]

Application Example: Image Stitching

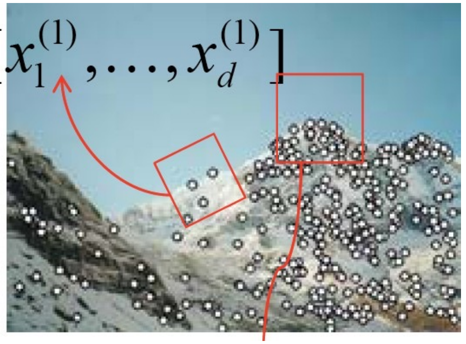
- Detection: Identify the interest points.

[Source: K. Grauman]

Application Example: Image Stitching

- Detection: Identify the interest points.
- Description: Extract feature vector descriptor around each interest point.

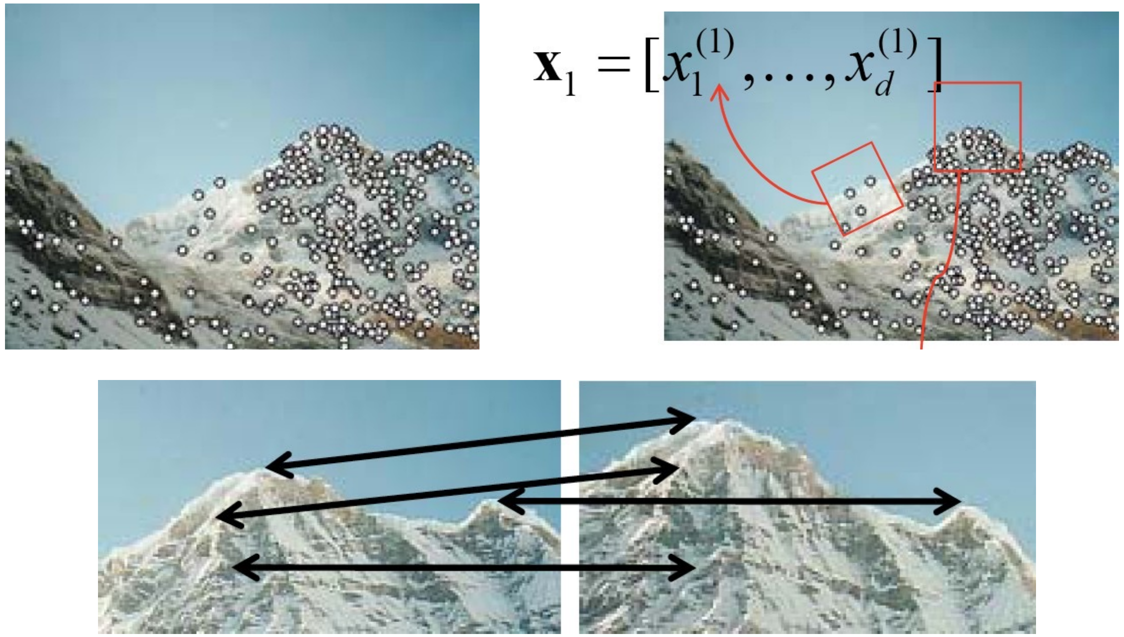
$$\mathbf{x}_1 = [x_1^{(1)}, \ldots, x_d^{(1)}]$$



[Source: K. Grauman]

Application Example: Image Stitching

- Detection: Identify the interest points.
- Description: Extract feature vector descriptor around each interest point.
- Matching: Determine correspondence between descriptors in two views.



[Source: K. Grauman]

Goal: Repeatability of the Interest Point Operator

- Our goal is to detect (at least some of) the same points in both images
- We need to run the detection procedure independently per image
- We need to generate enough points to increase our chances of detecting matching points
- We shouldn't generate too many or our matching algorithm will be too slow

Figure: Too few keypoints → little chance to find the true matches

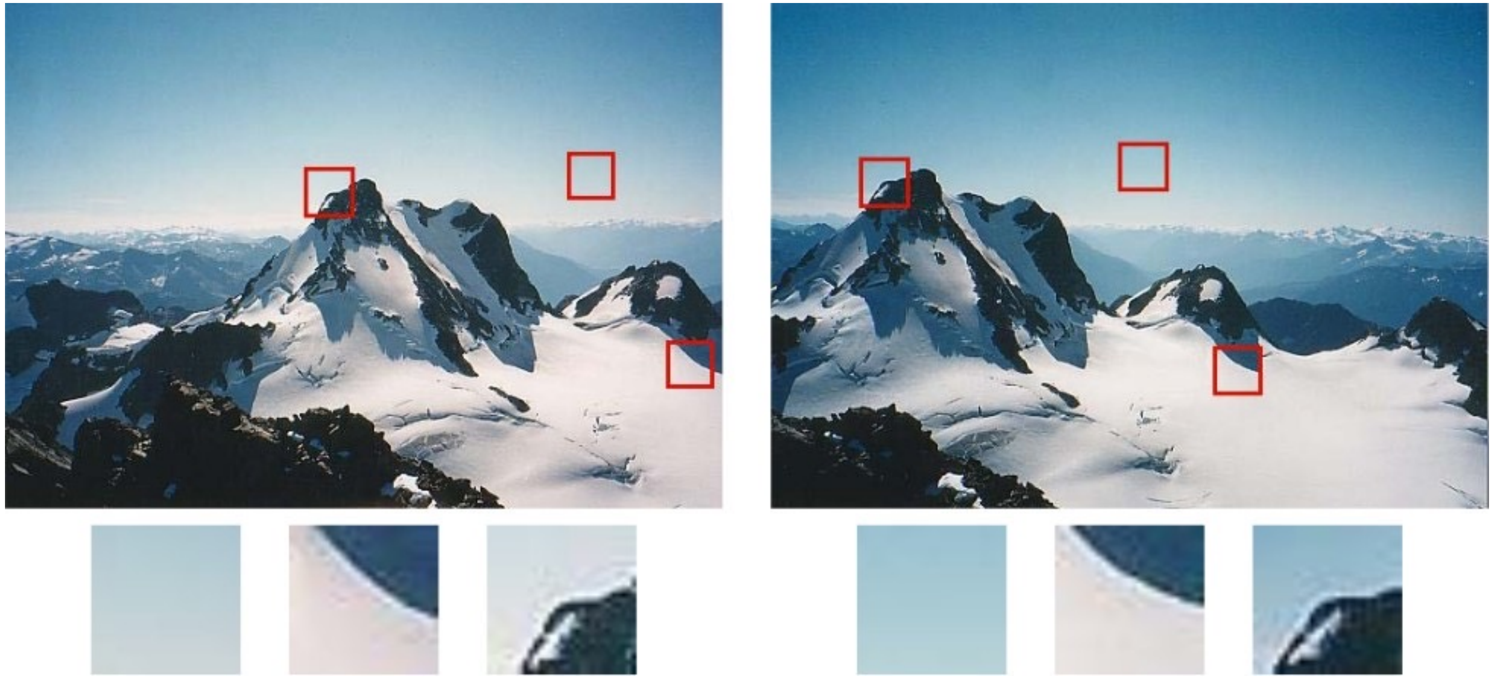
[Source: K. Grauman, slide credit: R. Urtasun]

What Points to Choose?

[Source: K. Grauman]

What Points to Choose for matching?

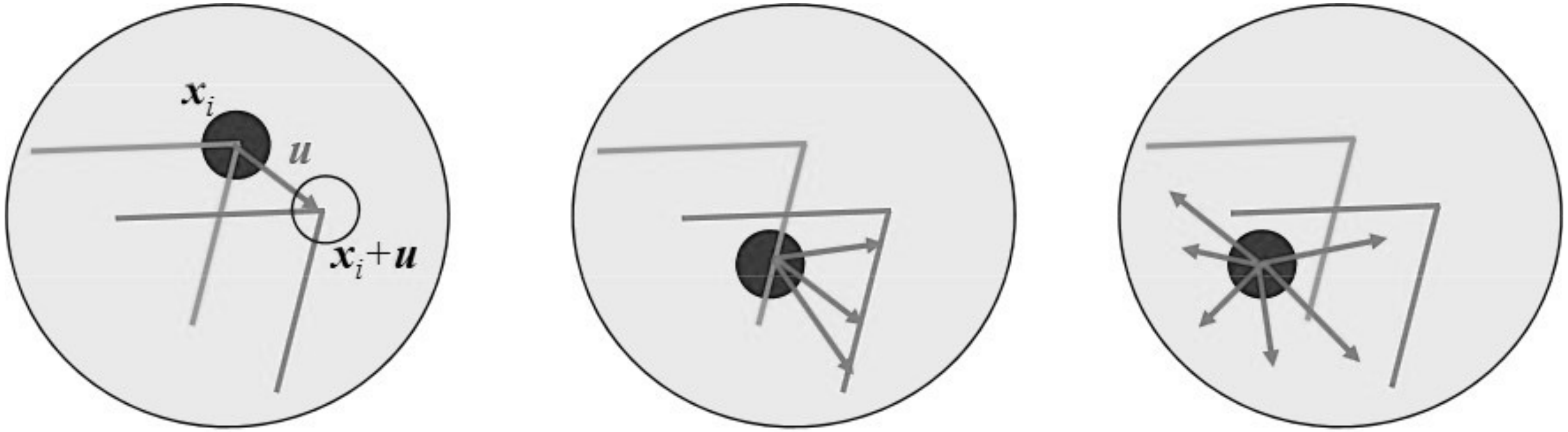
What Points to Choose for matching?



- Textureless patches are nearly impossible to localize.
- Patches with large contrast changes (gradients) are easier to localize.
- But straight line segments cannot be localized on lines segments with the same orientation (aperture problem)
- Gradients in at least two different orientations are easiest, e.g., corners!

[Adopted from: Szelski (Book)]

Aperture Problem



(a) (b) (c)

- "Corner-like" patch can be reliably matched
- A straight line patch can have multiple matches (Aperture Problem)
- Zero texture, useless, can have infinite matches

[Source: K. Grauman]

Corner Detection

Interest Points: Corners

- How can we find corners in an image?

Interest Points: Corners

- We should easily recognize the point by looking through a small window.
- Shifting a window in any direction should give a large change in intensity.

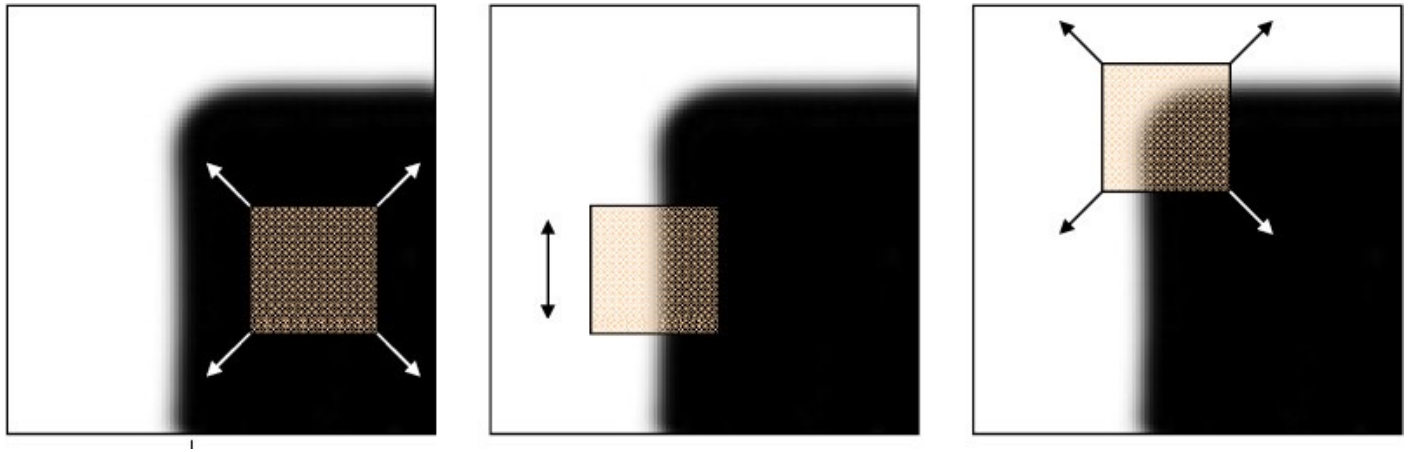


Figure: (left) flat region: no change in all directions, (center) edge: no change along the edge direction, (right) corner: significant change in all directions

[Source: Alyosha Efros, Darya Frolova, Denis Simakov]

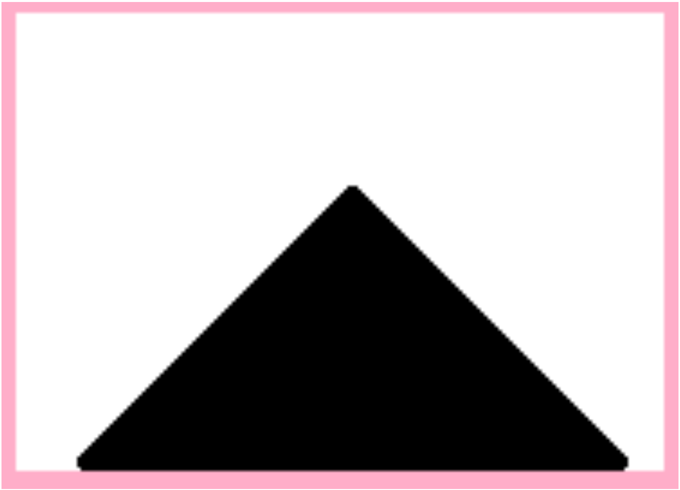
Interest Points: Corners

- Harris Corner Detector: Idea

$\sum I_x^2$ is large

$\sum I_y^2$ is large

Interest Points: Corners



$$\begin{bmatrix} \Sigma I_x^2 & \Sigma I_x I_y \\ \Sigma I_x I_y & \Sigma I_y^2 \end{bmatrix}$$

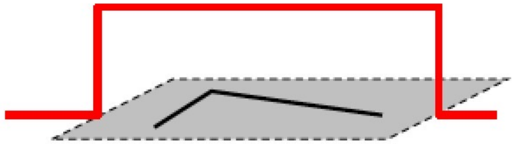
eigenvalues

Interest Points: Corners

- Compare two image patches using (weighted) summed square difference
- Measures change in appearance of window w(x, y) for the shift

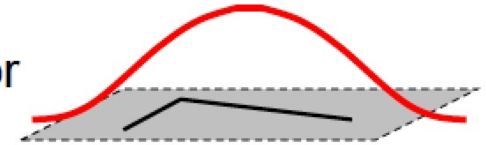
$$E_{\mathrm{WSSD}}(u,v) = \sum_x \sum_y \underbrace{w(x,y)}_{\text{window function}} [\underbrace{I(x+u,y+v)}_{\text{shifted intensity}} - \underbrace{I(x,y)}_{\text{intensity}}]^2$$

Window function $w(x,y) =$



1 in window, 0 outside

or



Gaussian

[Source: J. Hays]

Interest Points: Corners

- Compare two image patches using (weighted) summed square difference
- Measures change in appearance of window w(x, y) for the shift

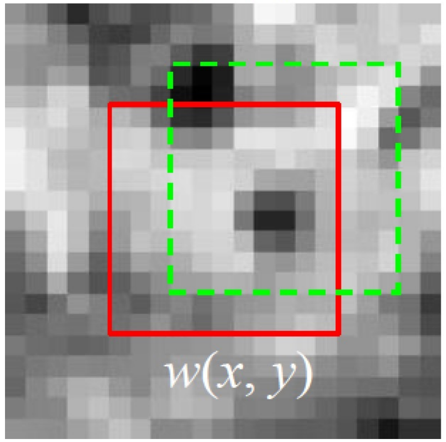
$$E_{\mathrm{WSSD}}(u, v) = \sum_x \sum_y w(x, y)[I(x+u, y+v) - I(x, y)]^2$$

window function

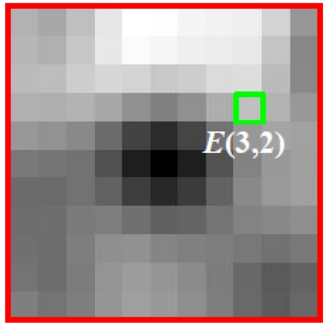
shifted intensity

intensity

$I(x, y)$



$E(u, v)$



[Source: J. Hays]

Interest Points: Corners

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$$E_{\mathrm{WSSD}}(u,v) = \sum_x \sum_y w(x,y)[I(x+u,y+v) - I(x,y)]^2$$

window function

shifted intensity

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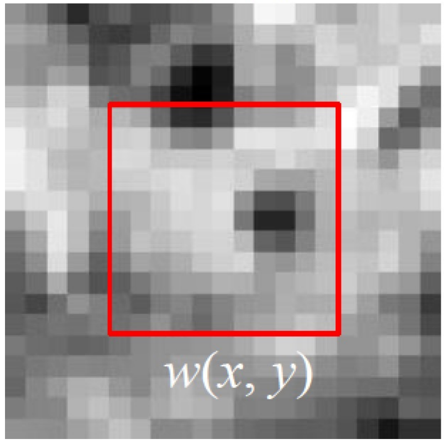


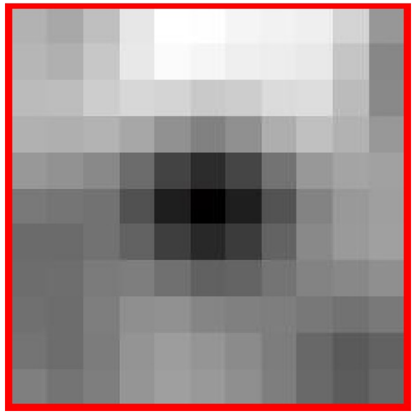
$E(u, v)$

[Source: J. Hays]

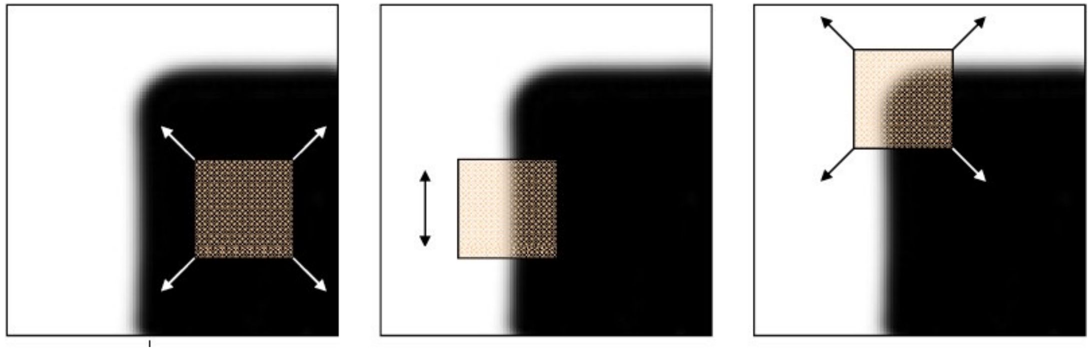
Interest Points: Corners

- Let's look at E_{WSSD}
- We want to find out how this function behaves for small shifts

$$E(u, v)$$



- Remember our goal to detect corners:



Interest Points: Corners

- Using a simple first order Taylor series expansion about *x, y* :

$$I(x+u, y+v) \approx I(x,y) + u \cdot \frac{\partial I}{\partial x}(x,y) + v \cdot \frac{\partial I}{\partial y}(x,y)$$

- Using a series of polynomials to approximate I, more info on Taylor Series here
- And plugging it in our expression for E_{WSSD}:

$$\begin{aligned}
E_{\text{WSSD}}(u,v) &= \sum_x \sum_y w(x,y)\Big(I(x+u,y+v) - I(x,y)\Big)^2 \\
&\approx \sum_x \sum_y w(x,y)\Big(I(x,y) + u \cdot I_x + v \cdot I_y - I(x,y)\Big)^2 \\
&= \sum_x \sum_y w(x,y)\Big(u^2 I_x^2 + 2u \cdot v \cdot I_x \cdot I_y + v^2\, I_y^2\Big) \\
&= \sum_x \sum_y w(x,y) \cdot \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} I_x^2 & I_x \cdot I_y \\ I_x \cdot I_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}
\end{aligned}$$

Interest Points: Corners

- Since (u, v) doesn't depend on (x, y) we can rewrite it slightly:

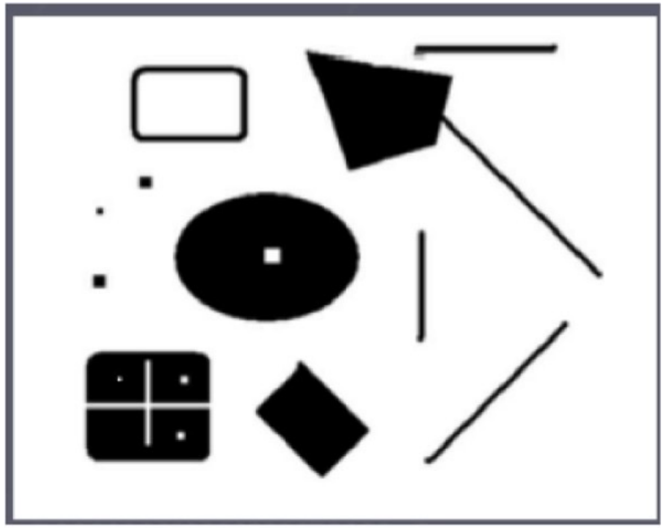
$$\begin{aligned} E_{\text{WSSD}}(u,v) &= \sum_x \sum_y w(x,y) \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} I_x^2 & I_x \cdot I_y \\ I_x \cdot I_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \\ &= \begin{bmatrix} u & v \end{bmatrix} \underbrace{\left(\sum_x \sum_y w(x,y) \begin{bmatrix} I_x^2 & I_x \cdot I_y \\ I_x \cdot I_y & I_y^2 \end{bmatrix} \right)}_{\text{Let's denotes this with } M} \begin{bmatrix} u \\ v \end{bmatrix} \\ &= \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix} \end{aligned}$$

- M is a 2x2 second moment matrix computed from image gradients

$$M = \sum_x \sum_y w(x,y) \begin{bmatrix} I_x^2 & I_x \cdot I_y \\ I_x \cdot I_y & I_y^2 \end{bmatrix}$$

How Do I Compute M ?

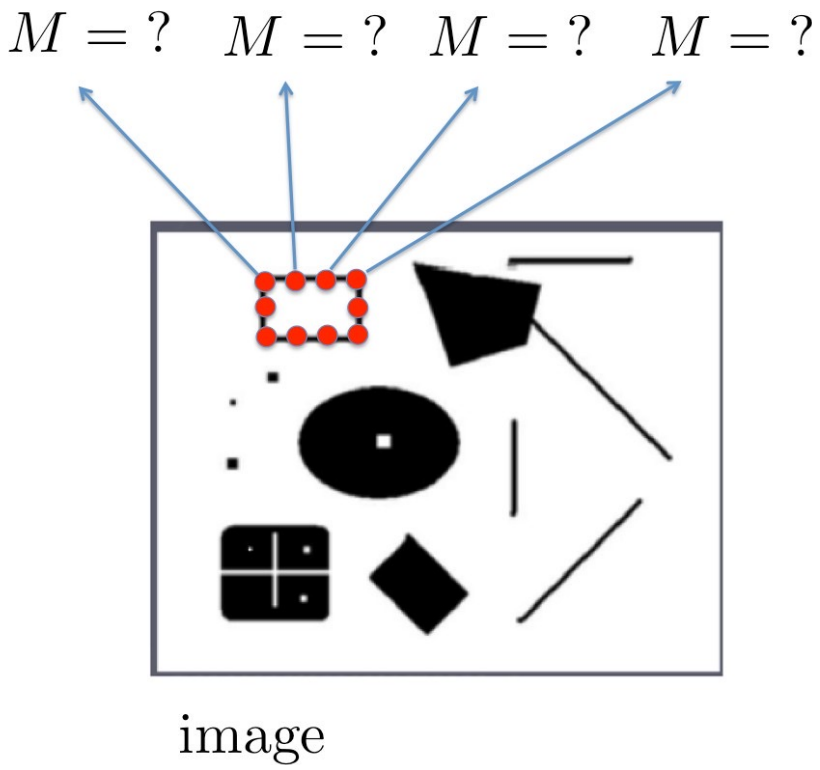
- Let’s say I have this image



image

How Do I Compute M ?

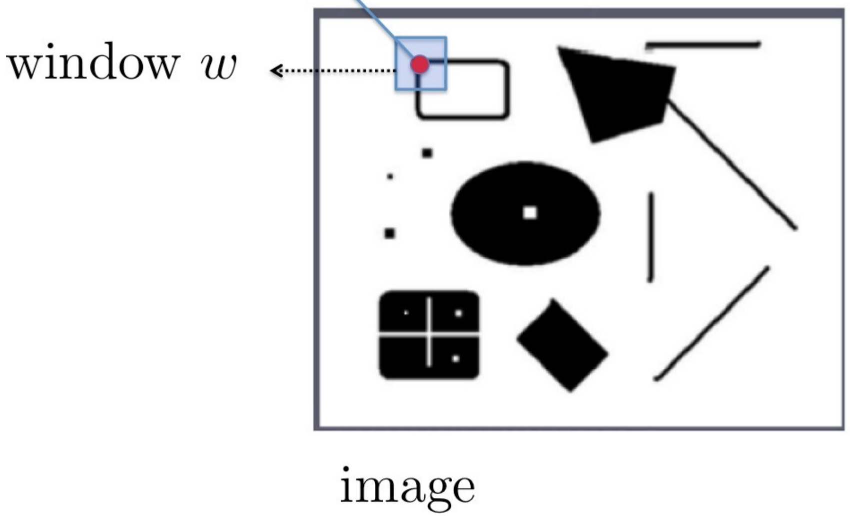
- Let's say I have this image
- I need to compute a 2 × 2 second moment matrix in each image location



How Do I Compute M ?

- Let's say I have this image
- I need to compute a 2 × 2 second moment matrix in each image location
- In a particular location I need to compute M as a weighted average of gradients in a window

$$M = \sum_x \sum_y w(x,y) \begin{bmatrix} I_x^2 & I_x \cdot I_y \\ I_x \cdot I_y & I_y^2 \end{bmatrix}$$



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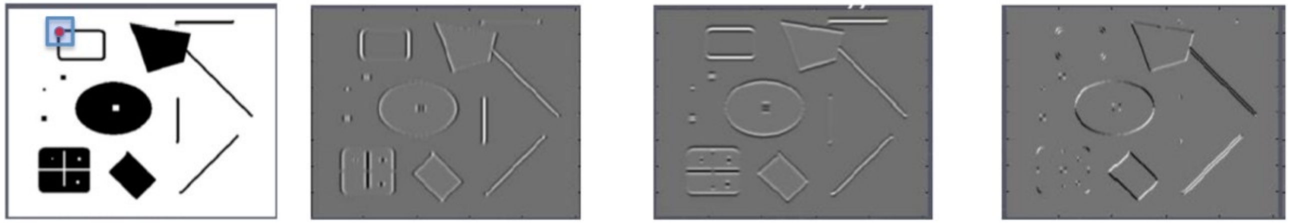


image $\quad I_x = \frac{\partial I}{\partial x} \quad I_y = \frac{\partial I}{\partial y} \quad I_x \cdot I_y$

- Let's say I have this image
- I need to compute a 2 × 2 second moment matrix in each image location
- In a particular location I need to compute M as a weighted average of gradients in
- a window

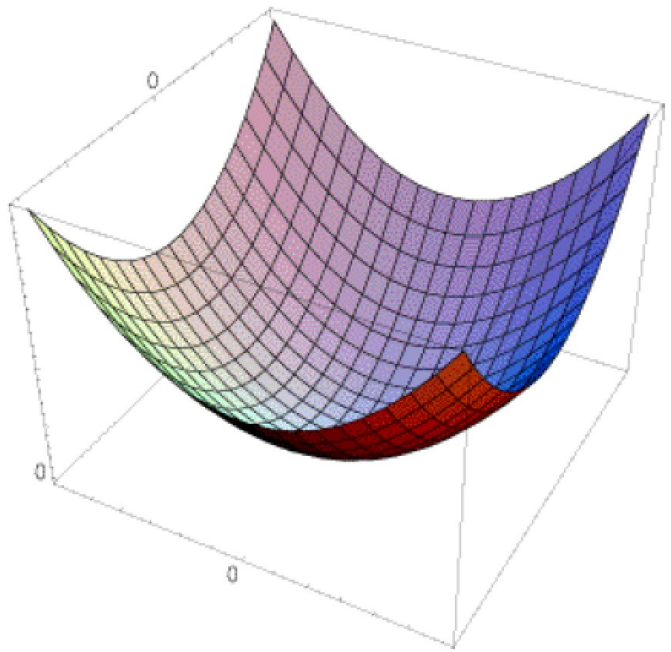
I can do this efficiently by computing three matrices, I_x^2 , I_y^2 and $I_x \cdot I_y$, and convolving each one with a filter, e.g. a box or Gaussian filter

How Do I Compute M ?

- We now have *M* computed in each image location
- Our E_{WSSD} is a quadratic function where *M* implies its shape

$$E_{\text{WSSD}}(u,v) \quad = \quad \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum_x \sum_y w(x,y) \begin{bmatrix} I_x^2 & I_x \cdot I_y \\ I_x \cdot I_y & I_y^2 \end{bmatrix}$$



[Source: J. Hays]

How Do I Compute M ?

- Let's take a horizontal "slice" of $E_{WSSD}(u, v)$:

$$\begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$

How Do I Compute M ?

- Let's take a horizontal "slice" of $E_{WSSD}(u, v)$:

$$\begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$

- This is the equation of an ellipse

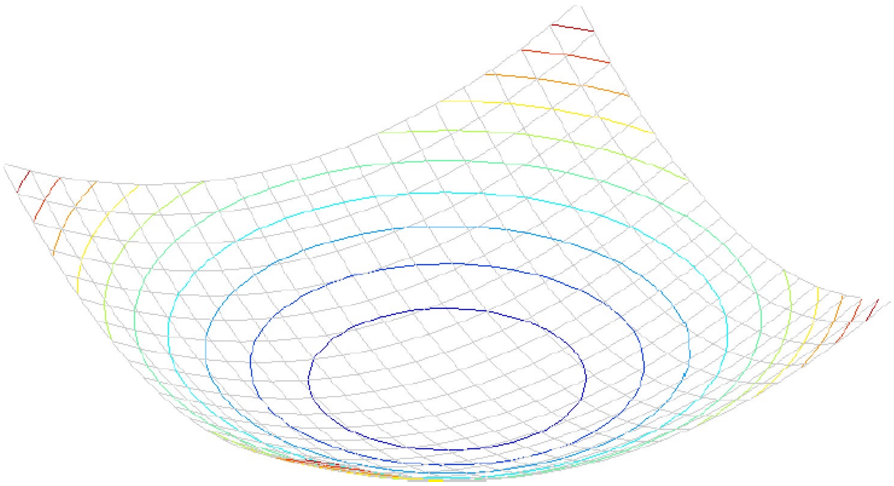


Figure: Different ellipses obtain by different horizontal "slices"

How Do I Compute M ?

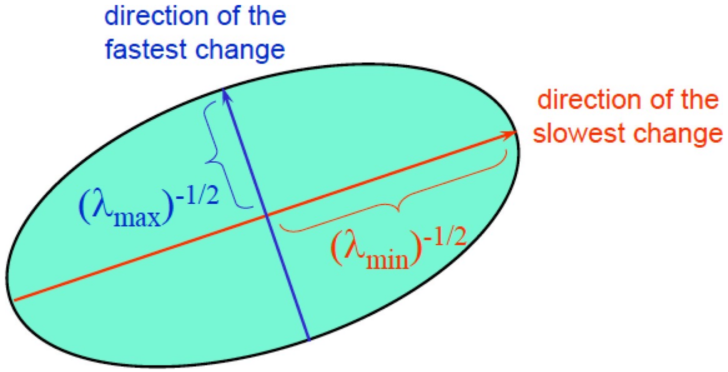
- Our matrix M is symmetric:

$$M = \sum_x \sum_y w(x,y) \begin{bmatrix} I_x^2 & I_x \cdot I_y \\ I_x \cdot I_y & I_y^2 \end{bmatrix}$$

- And thus we can diagonalize it (in Matlab: [V,D] = eig(M)):

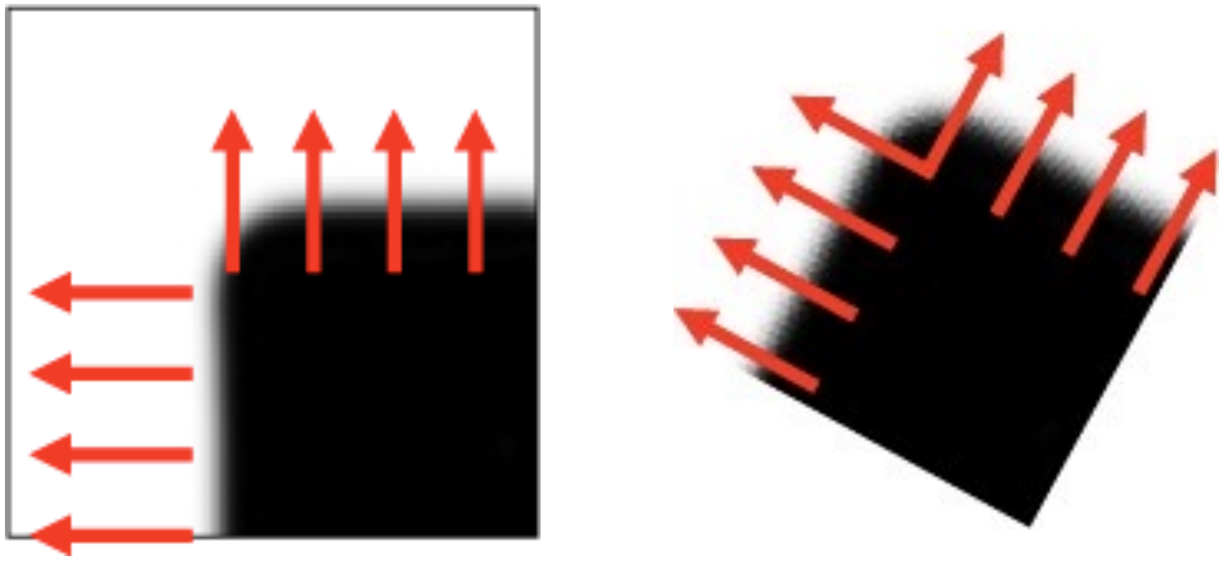
$$M = V \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} V^{-1}$$

- Columns of V are major and minor axes of ellipse, the lengths of the radii proportional to $\lambda^{-1/2}$



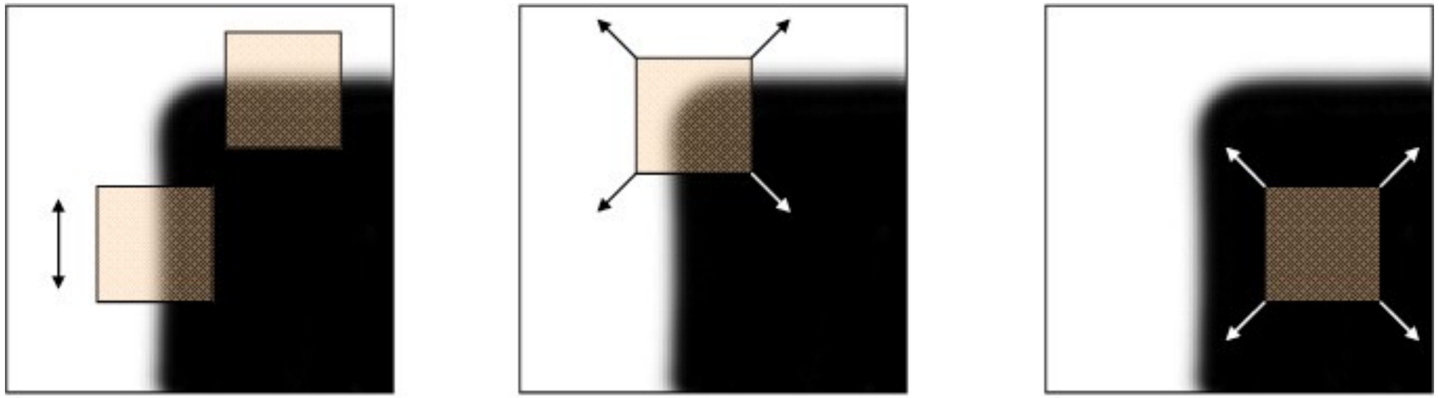
How Do I Compute M ?

- The eigenvalues of M (λ_1, λ_2) reveal the amount of intensity change in the two principal orthogonal gradient directions in the window



[Source: R. Szeliski, slide credit: R. Urtasun]

How Do I Compute M ?



"edge":
$\lambda_1 >> \lambda_2$
$\lambda_2 >> \lambda_1$

"corner":
λ_1 and λ_2 are large,
$\lambda_1 \sim \lambda_2$;

"flat" region
λ_1 and λ_2 are small;

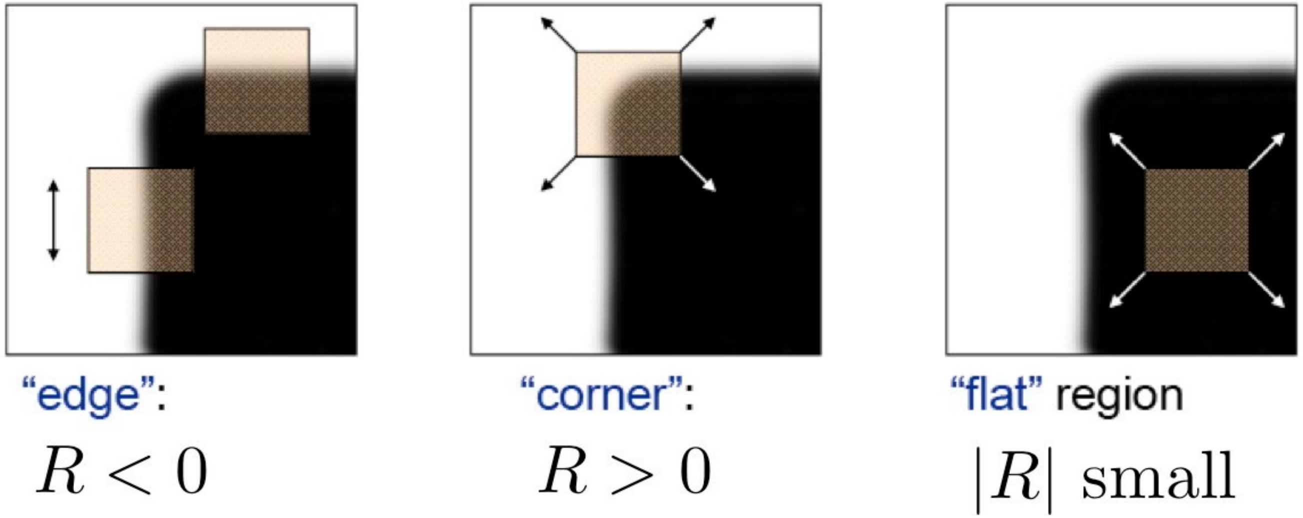
[Source: K. Grauman, slide credit: R. Urtasun]

Interest Points: Criteria to Find Corners

- Harris and Stephens, '88, is rotationally invariant and downweighs edge-like features where $\lambda_1 \gg \lambda_0$

$$R = \lambda_0\lambda_1 - \alpha(\lambda_0 + \lambda_1)^2 = \mathrm{det}(M) - \alpha \cdot \mathrm{trace}(M)^2$$

- **Why** go via det and trace and not use a criteria with λ?
- α a constant (0.04 to 0.06)



- The corresponding detector is called Harris corner detector

Interest Points: Criteria to Find Corners

- Harris & Stephens (1998)

$$R = \lambda_0\lambda_1 - \alpha(\lambda_0 + \lambda_1)^2 = \mathrm{det}(M) - \alpha \cdot \mathrm{trace}(M)^2$$

- Kande & Tomasi (1994)

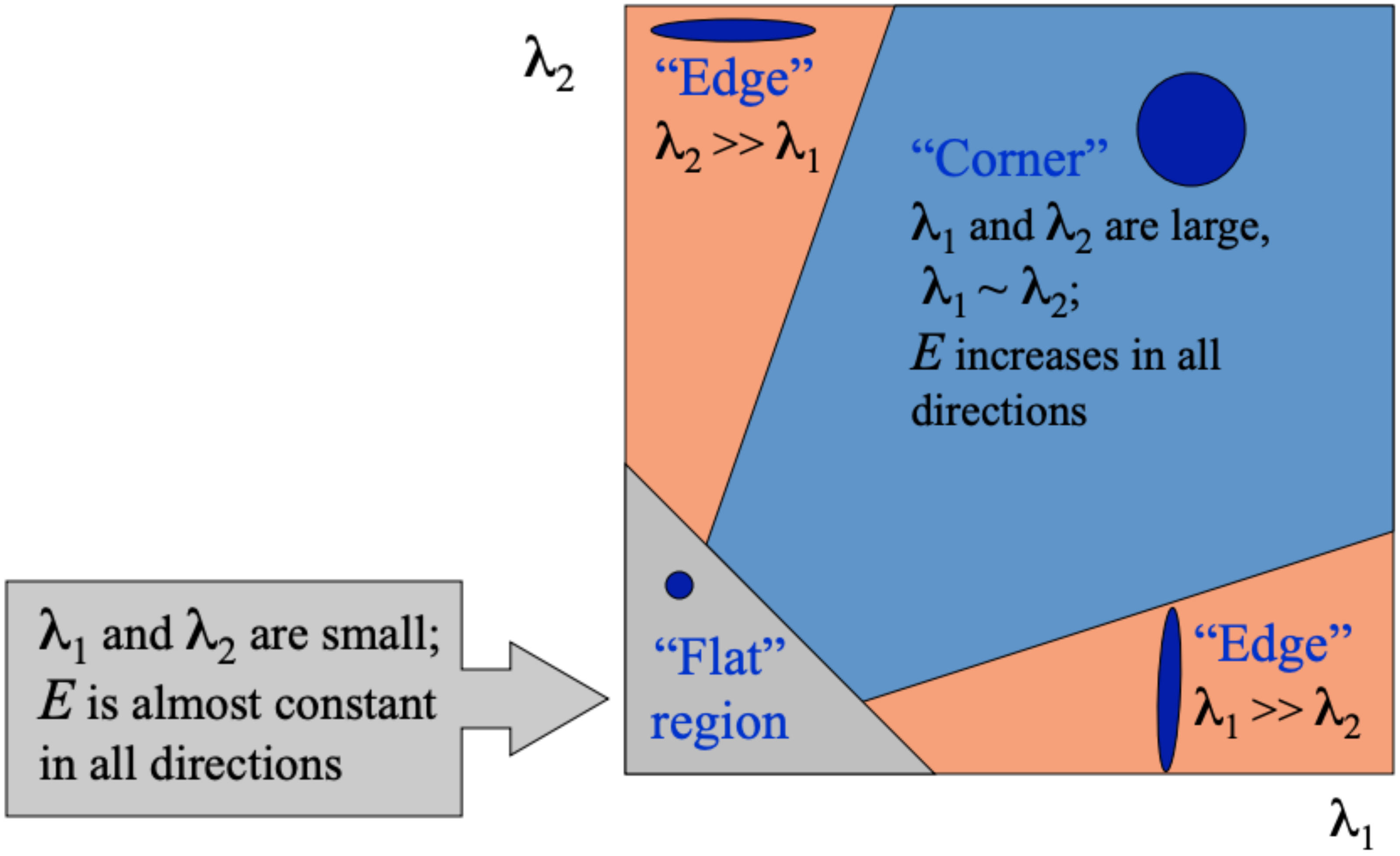
$$R = \min(\lambda_1, \lambda_2)$$

- Nobel (1998)

$$R = \frac{\det(M)}{\mathrm{trace}(M)+\epsilon}$$

[Source Mubarak Shah, Szelski]

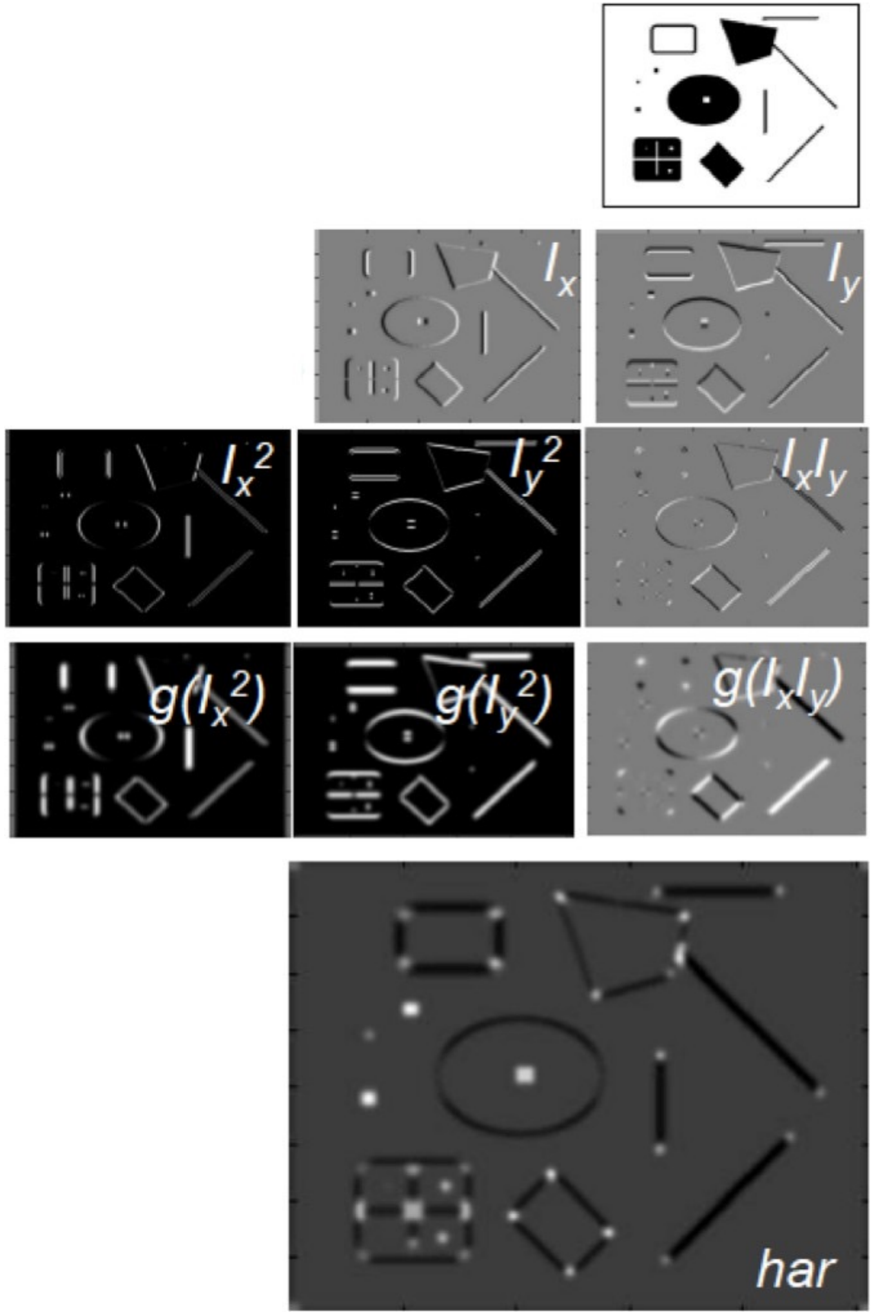
Interest Points: Criteria to Find Corners



[Source: K. Bala]

Harris Corner detector

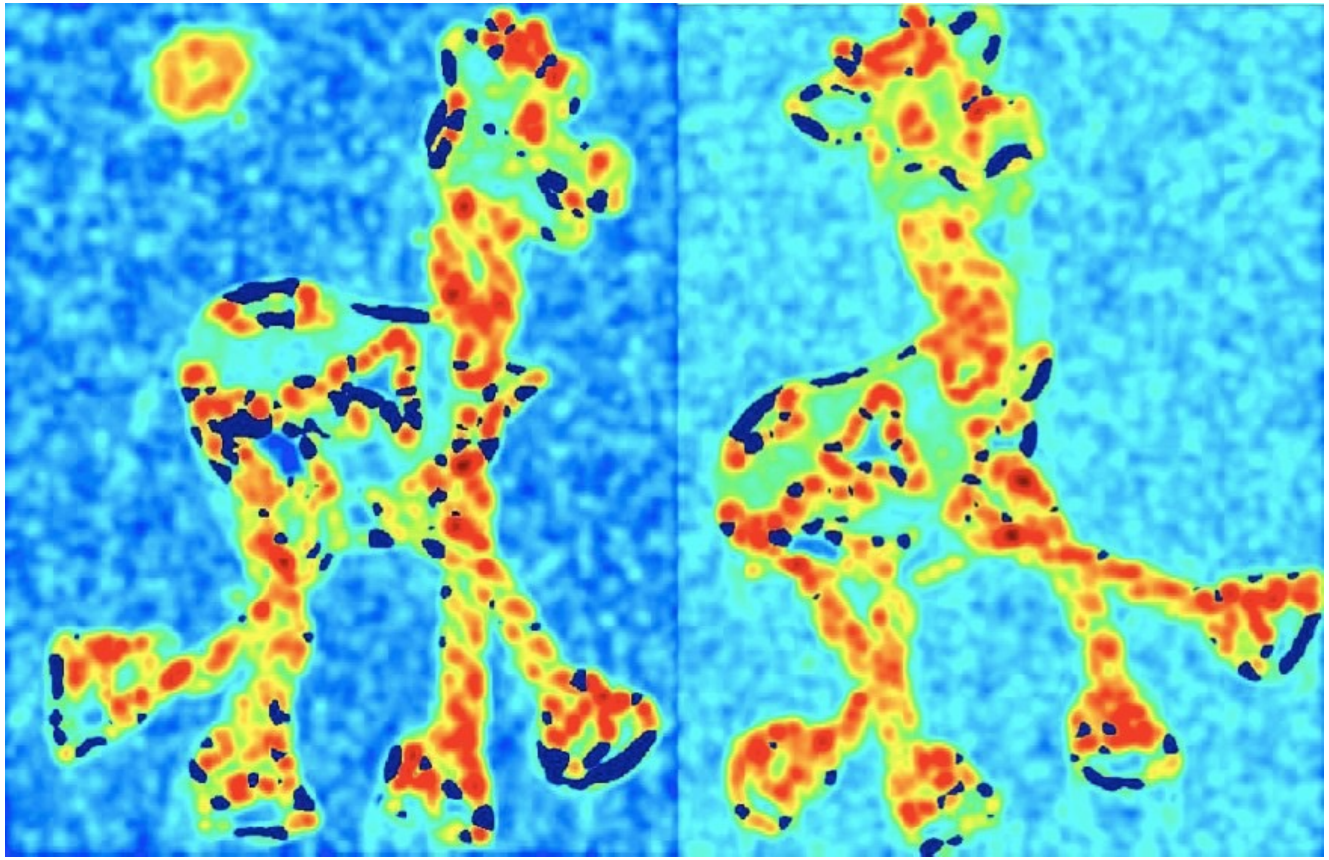
- Compute gradients I_x and I_y
- Compute I_x^2, I_y^2 , $I_x \cdot I_y$
- Average (Gaussian) → gives M per voxel
- Compute $R = \det(M) - \alpha\,\mathrm{trace}(M)^2$ for each image window (cornerness score)
- Find points with large R ($R >$ threshold).
- Take only points of local maxima, i.e., perform non-maximum suppression



Example

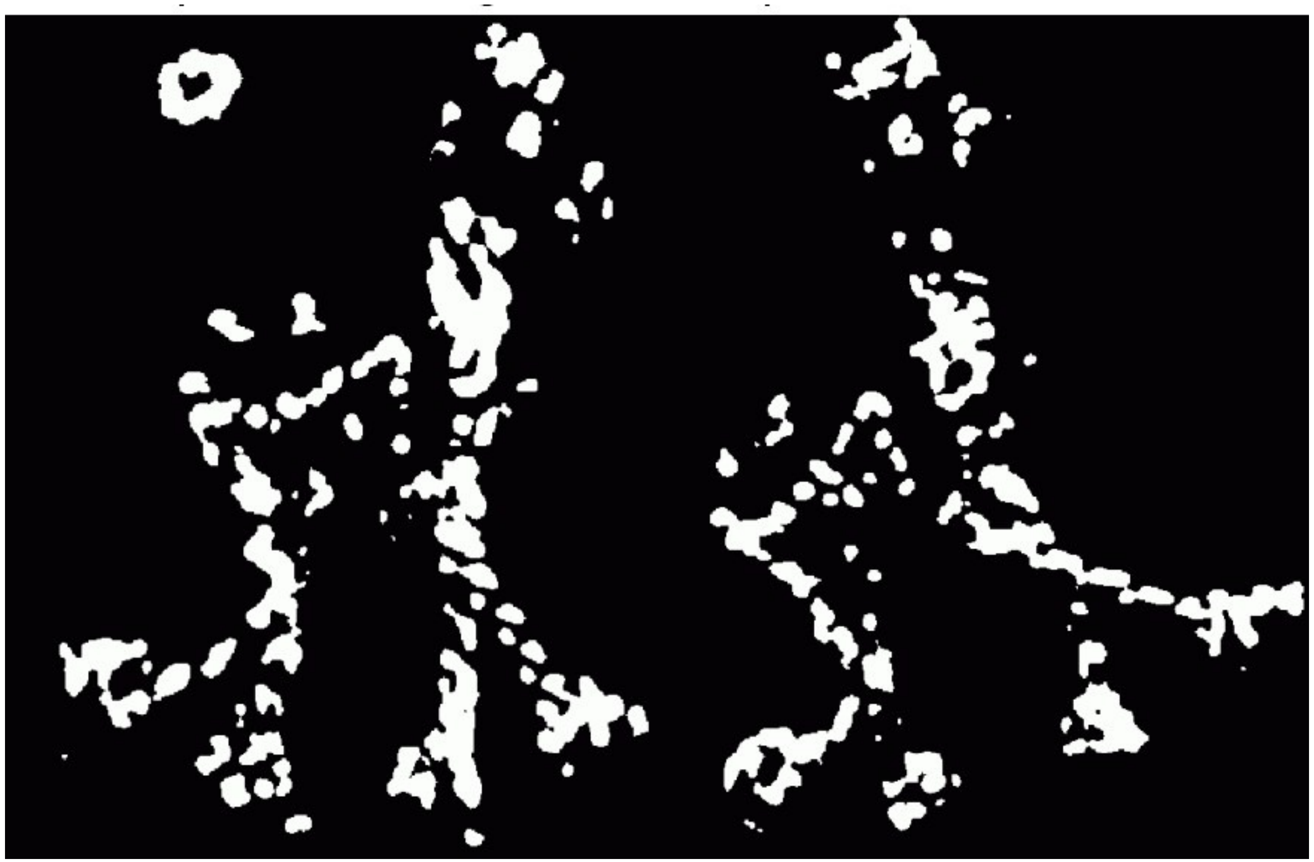
[Source: K. Grauman]

1) Compute Cornerness



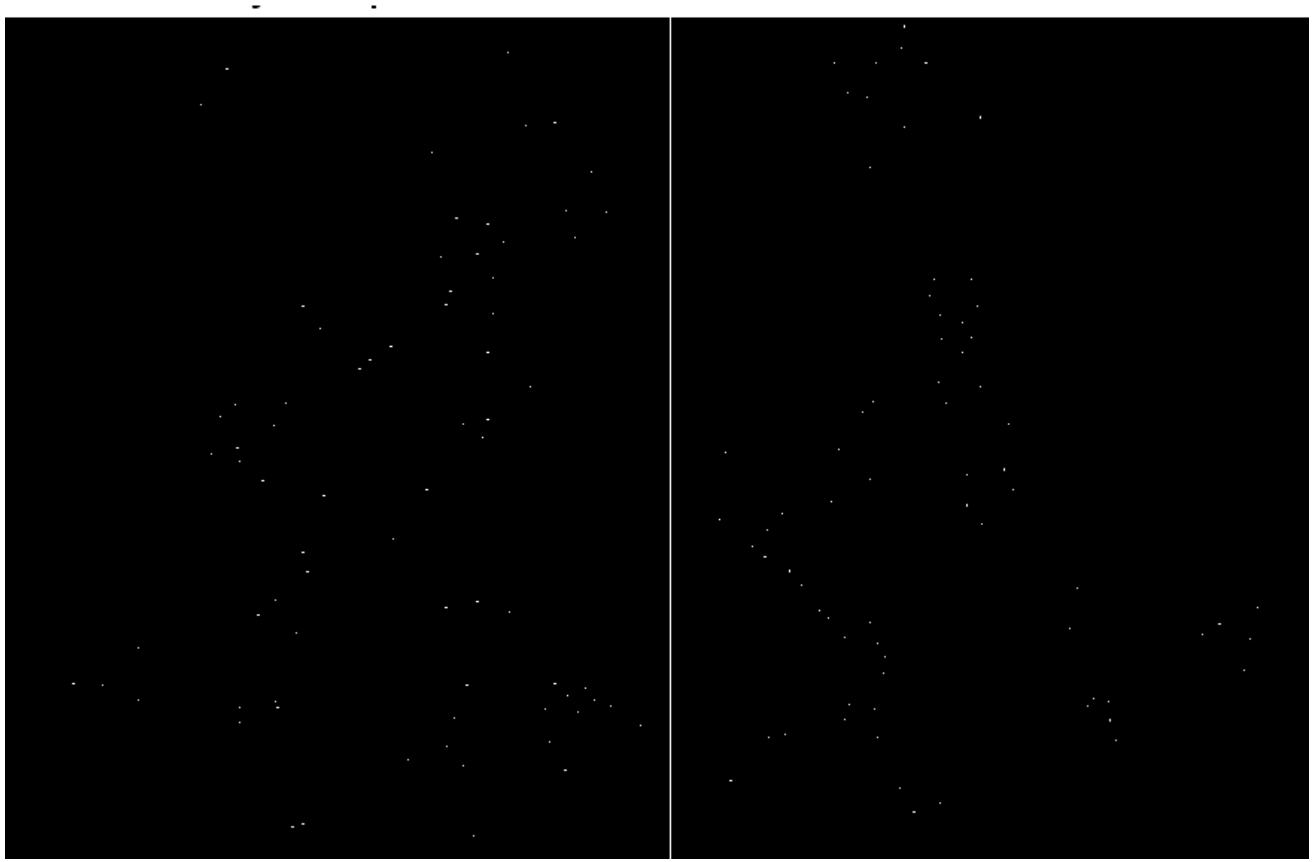
[Source: K. Grauman]

2) Find High Response



[Source: K. Grauman]

3) Non-maxima Suppresion



[Source: K. Grauman]

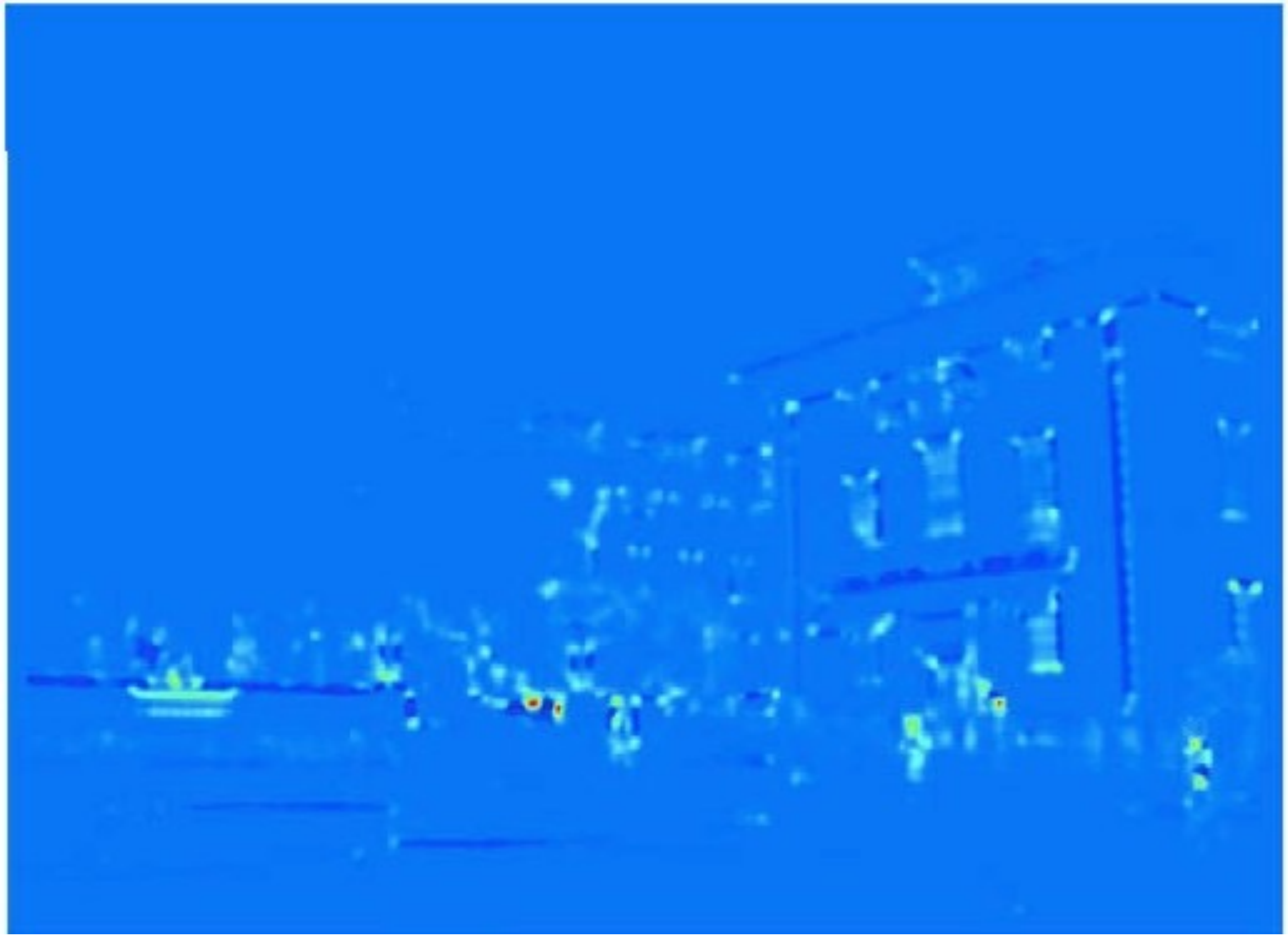
Results

[Source: K. Grauman]

Another Example

[Source: K. Grauman]

Cornerness



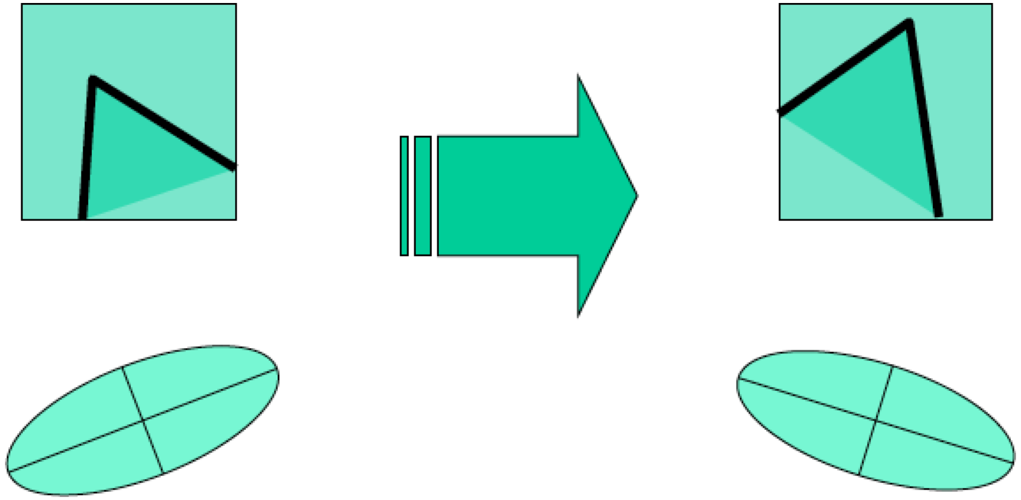
[Source: K. Grauman]

Interest Points

[Source: K. Grauman]

Properties of Harris Corner Detector

- Rotation and Shift Invariance of Corners

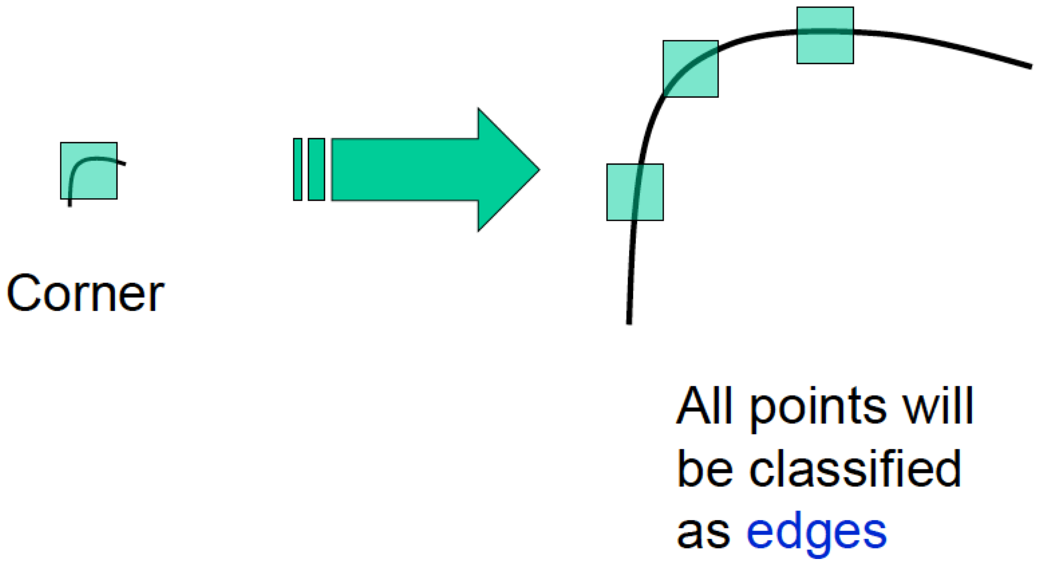


- Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same
- Harris corner detector is rotation-covariant

[Source: J. Hays]

Properties of Harris Corner Detector

- Scale?



- Corner location is not scale invariant/covariant!

[Source: J. Hays]

Optical Flow

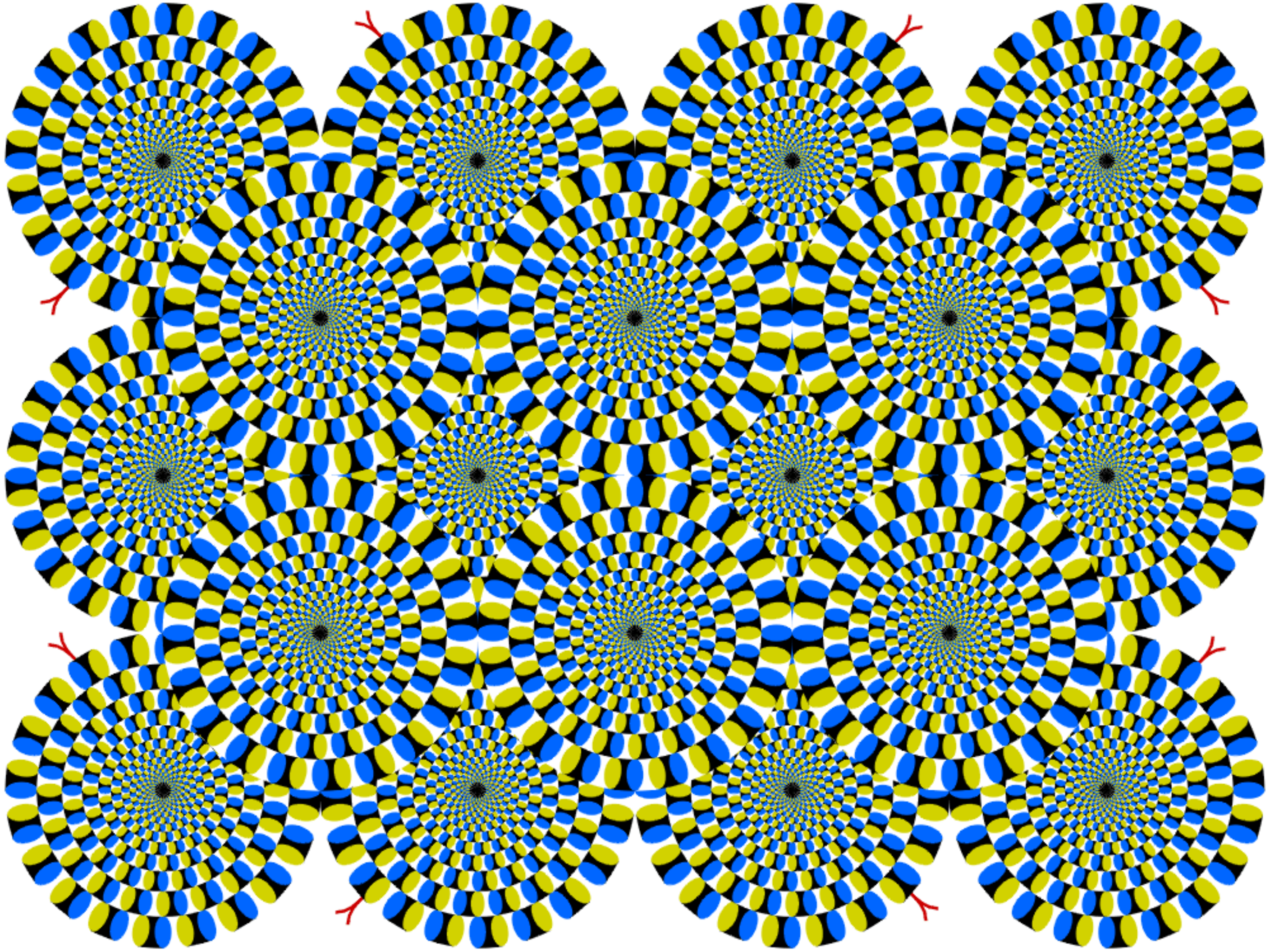
Slide Credit: Ali Farhadi

We live in a moving world

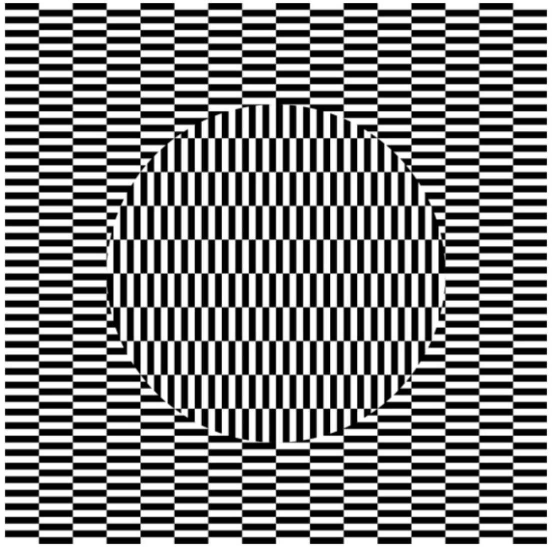
- Perceiving, understanding and predicting motion is an important part of our daily lives

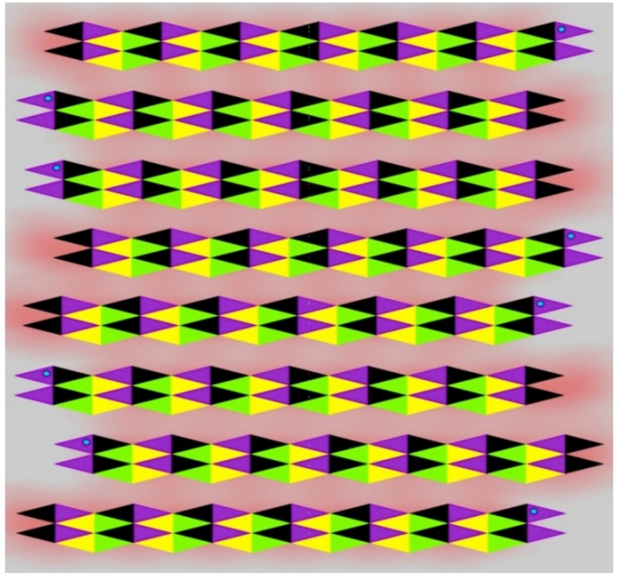
Jonschkowski et al. 2020]

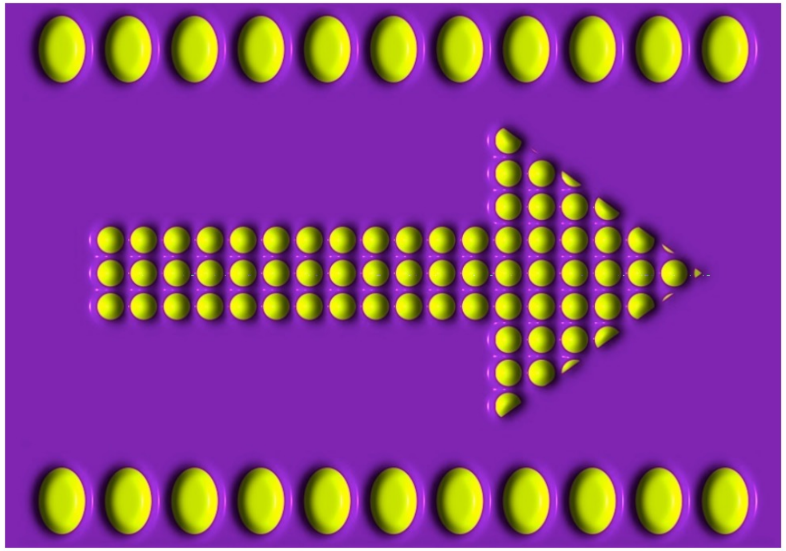
Seeing motion from a static picture?

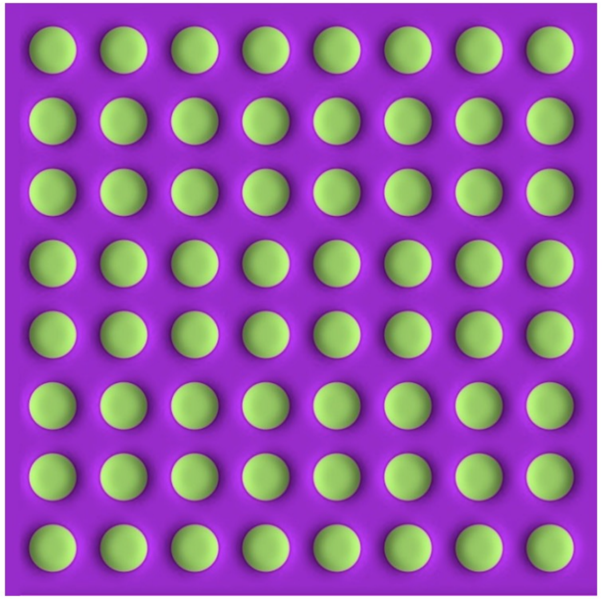


More examples







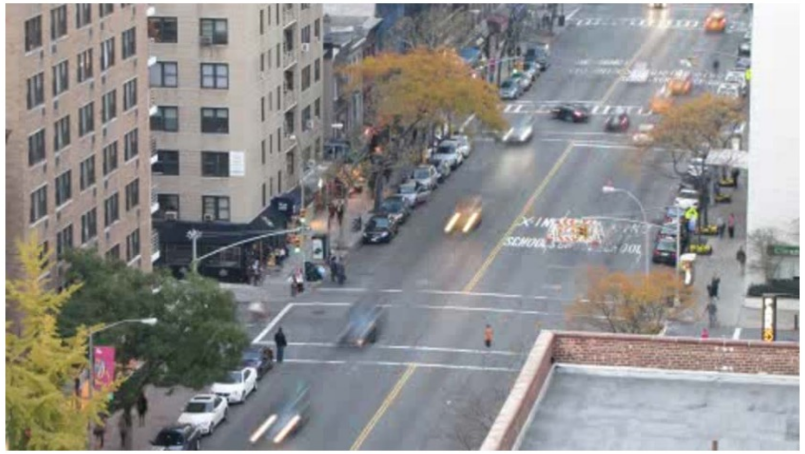


Motion scenarios (priors)

Static camera, moving scene

Moving camera, static scene

Moving camera, moving scene



Static camera, moving scene, moving light

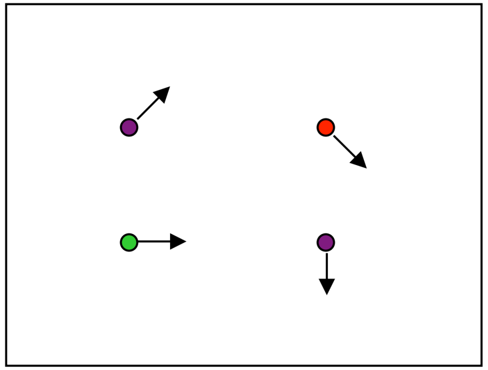
How can we recover motion?

- Extract visual features (corners, textured areas) and “track” them over multiple frames.

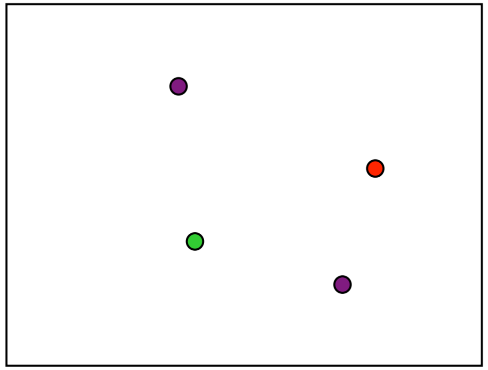
- Recover image motion at each pixel from spatio-temporal image brightness variations (optical flow).

B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In Proceedings of the International Joint Conference on Artificial Intelligence, pp. 674–679, 1981.

Feature tracking



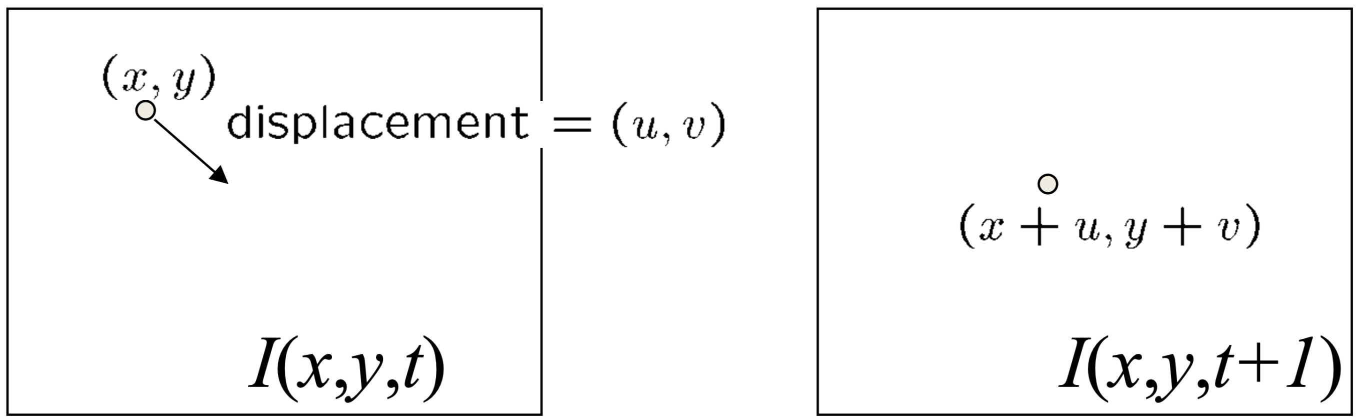
$I(x,y,t)$



$I(x,y,t+1)$

- Given two subsequent frames, estimate the point translation
- Key assumptions:
  - Brightness constancy: projection of the same point looks the same in every frame
  - Small motion: points do not move very far
  - Spatial coherence: points move like their neighbors

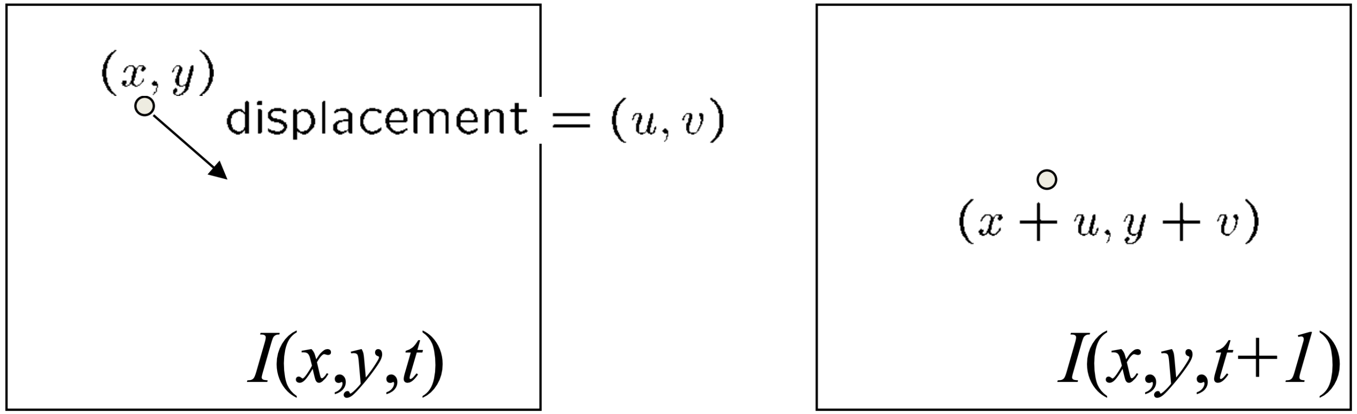
The brightness constancy constraint



- Brightness Constancy Equation: $I(x, y, t) = I(x + u\,, y + v, t + 1)$

- Now, take the Taylor expansion of $I(x + u\,, y + v, t + 1)$ at (x, y, t) to linearize the right side

The brightness constancy constraint



Brightness Constancy Equation: $I(x, y, t) = I(x + u\ , y + v, t + 1)$

$$I(x + u\ , y + v, t + 1) \ \approx I(x, y, t) + I_x . u + I_y . v + I_t$$

$$I(x + u\ , y + v, t + 1) - I(x, y, t) \approx +I_x . u + I_y . v + I_t$$

$$\nabla I \ \begin{bmatrix} u \\ v \end{bmatrix} + I_t = 0$$

The brightness constancy constraint

- Can we use this equation to recover image motion (u,v) at each pixel?

$$\nabla I \cdot \begin{bmatrix} u \\ v \end{bmatrix} + I_t = 0$$

- How many equations and unknowns per pixel?

The brightness constancy constraint

- Can we use this equation to recover image motion (u,v) at each pixel?

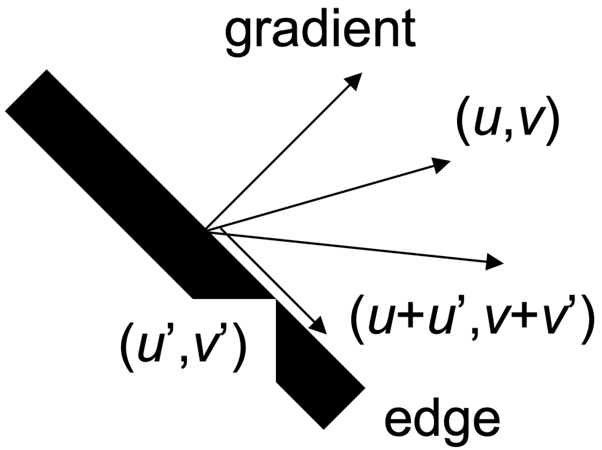
$$\nabla I \, . \begin{bmatrix} u \\ v \end{bmatrix} + I_t = 0$$

- How many equations and unknowns per pixel?
- One equation (this is a scalar equation!), two unknowns (u,v)

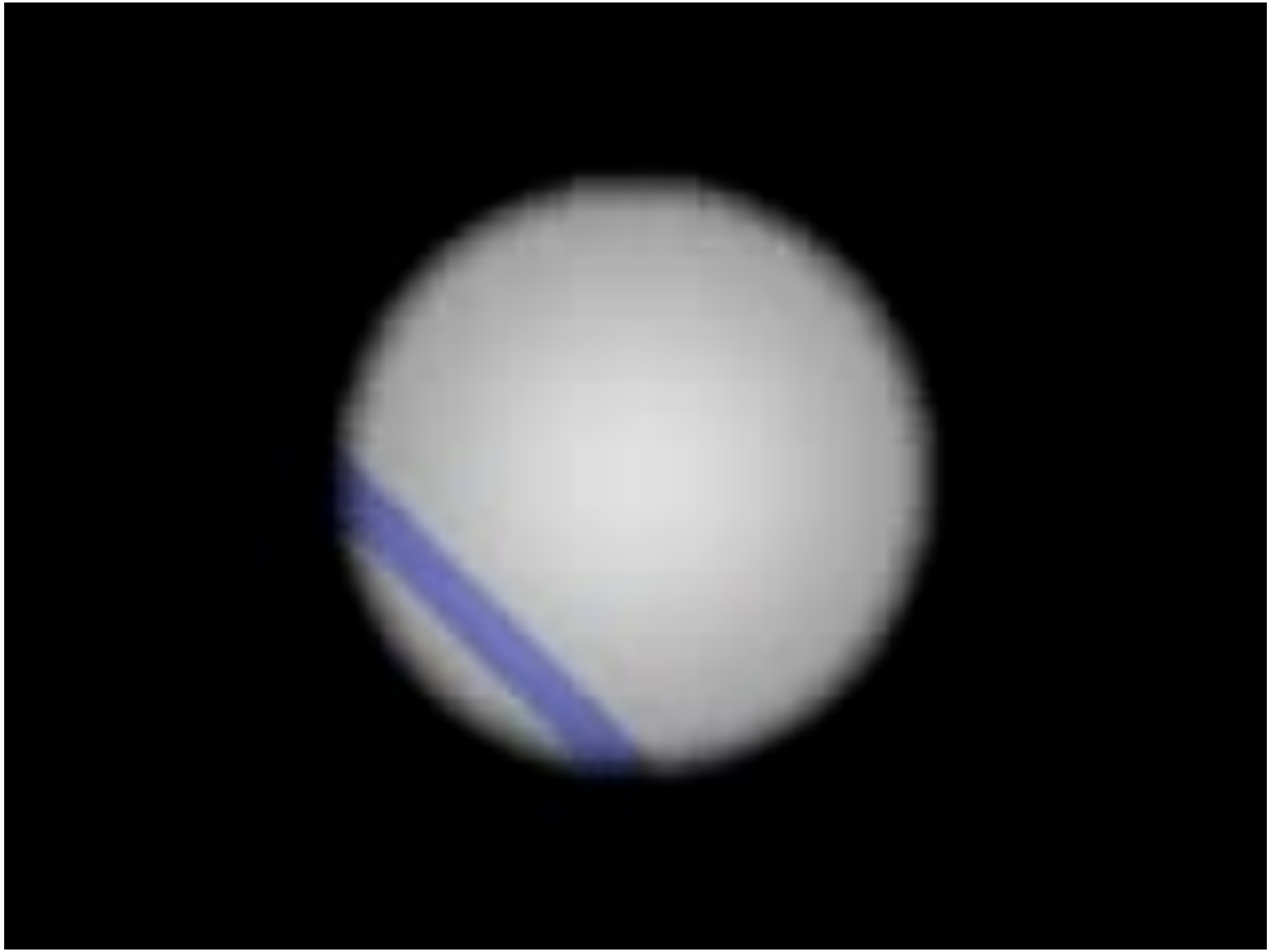
The brightness constancy constraint

- The component of the motion perpendicular to the gradient (i.e., parallel to the edge) cannot be measured.
  - If (u, v) satisfies the equation, so does $(u + u', v + v')$ if

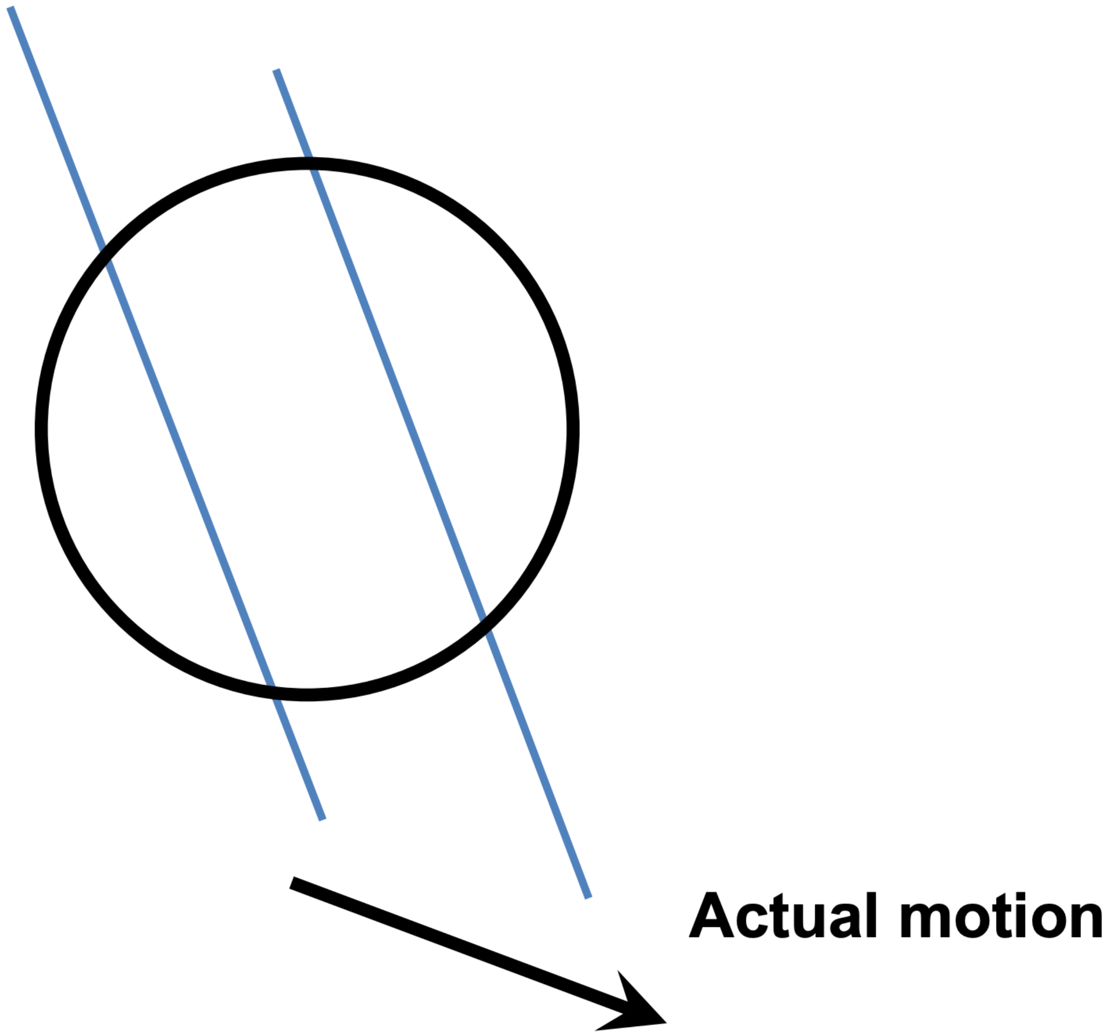
$$\nabla I \,.\begin{bmatrix} u' \\ v' \end{bmatrix} = 0$$



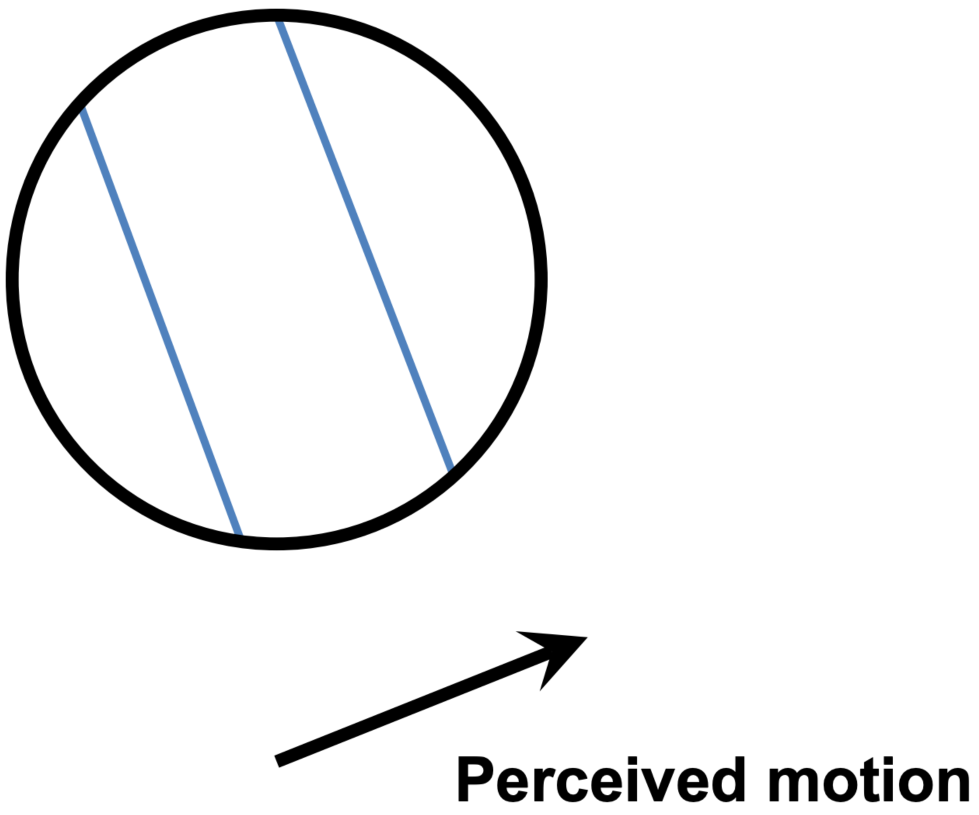
The aperture problem



The aperture problem



The aperture problem



The barber pole illusion

Solving the ambiguity…

- How to get more equations for a pixel?
- Spatial coherence constraint
- Assume the pixel's neighbors have the same (u, v)
    - If we use a 5x5 window, that gives us 25 equations per pixel

    - For $\forall_{p_i} : \nabla I(p_i). \begin{bmatrix} u \\ v \end{bmatrix} + I_t(p_i) = 0$

Solving the ambiguity...

$$\begin{pmatrix} I_x(p_1) & I_y(p_1) \\ \vdots & \vdots \\ I_x(p_{25}) & I_y(p_{25}) \end{pmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \begin{pmatrix} I_t(p_1) \\ \vdots \\ I_t(p_{25}) \end{pmatrix} = 0$$

$$\underbrace{\begin{pmatrix} I_x(p_1) & I_y(p_1) \\ \vdots & \vdots \\ I_x(p_{25}) & I_y(p_{25}) \end{pmatrix}}_{A} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{d} = \underbrace{-\begin{pmatrix} I_t(p_1) \\ \vdots \\ I_t(p_{25}) \end{pmatrix}}_{b}$$

$$A\,d = b$$

Solving the ambiguity...

- Least squares solution for d given by

$$A^T T d = A^T b$$

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

- The summations are over all pixels in the K x K window

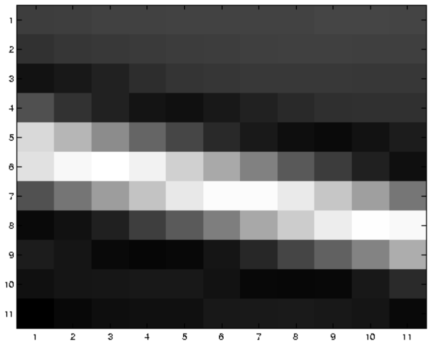
- Does this look familiar?

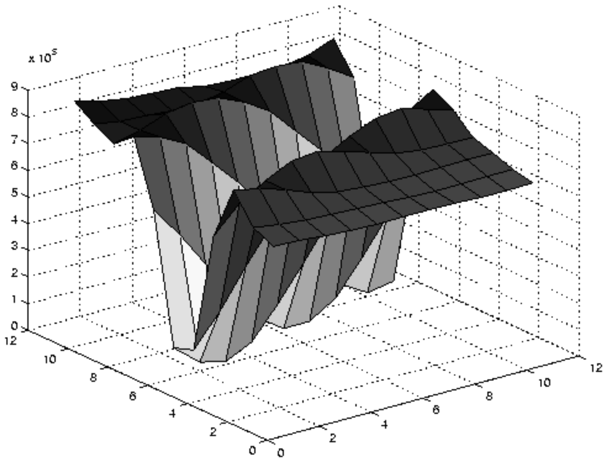
Conditions for solvability

- Optimal (u, v) satisfies Lucas-Kanade equation

- When is this solvable? I.e., what are good points to track?
  - A^TA should be invertible
  - A^TA should not be too small due to noise
    – eigenvalues λ_1 and λ_2 of A^TA should not be too small
  - A^TA should be well-conditioned
    – λ_1/λ_2 should not be too large (λ_1=larger eigenvalue)

Edges cause problems



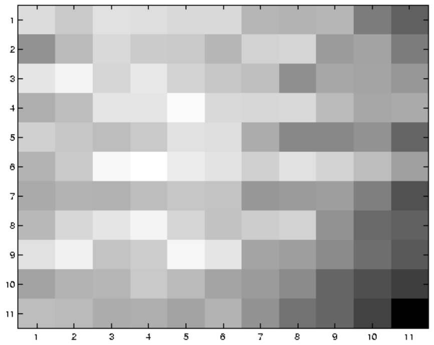


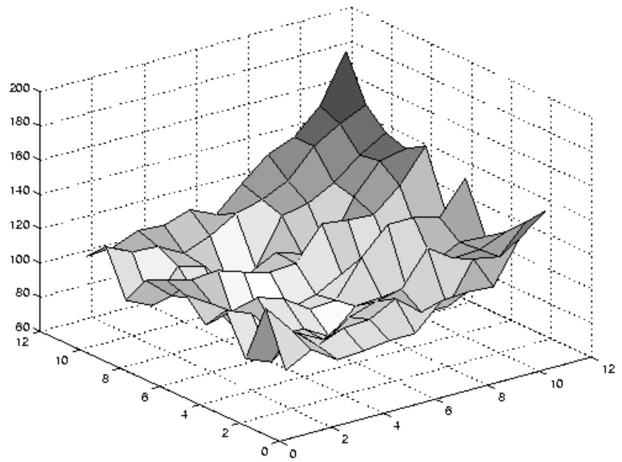
$$\sum \nabla I (\nabla I)^T$$

– large gradients, all the same

– large λ_1, small λ_2

Low texture regions don't work



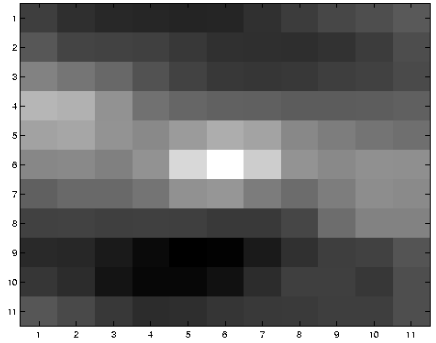


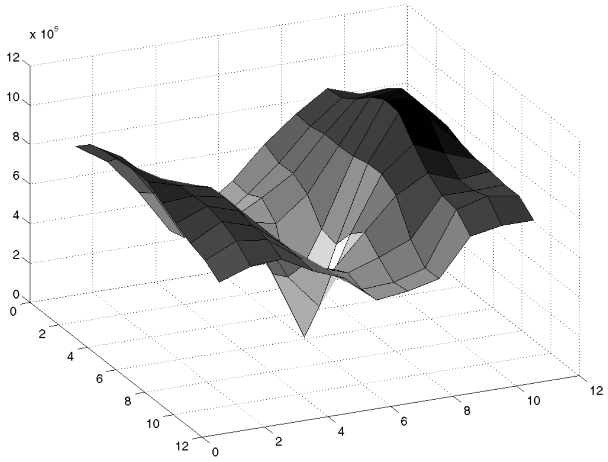
$$\sum \nabla I (\nabla I)^T$$

– gradients have small magnitude

– small λ_1, small λ_2

High textured region work best





$$\sum \nabla I (\nabla I)^T$$

– gradients are different, large magnitudes

– large λ_1, large λ_2

Errors in Lukas-Kanade

- What are the potential causes of errors in this procedure?
  - Suppose A^TA is easily invertible
  - Suppose there is not much noise in the image

When our assumptions are violated

- Brightness constancy is not satisfied
- The motion is not small
- A point does not move like its neighbors
  - window size is too large
  - what is the ideal window size?

Dealing with larger movements: Iterative refinement

Original (x,y) position

$$I_t = I(x', y', t+1) - I(x, y, t)$$

1. Initialize $(x', y') = (x, y)$
2. Compute (u, v) by

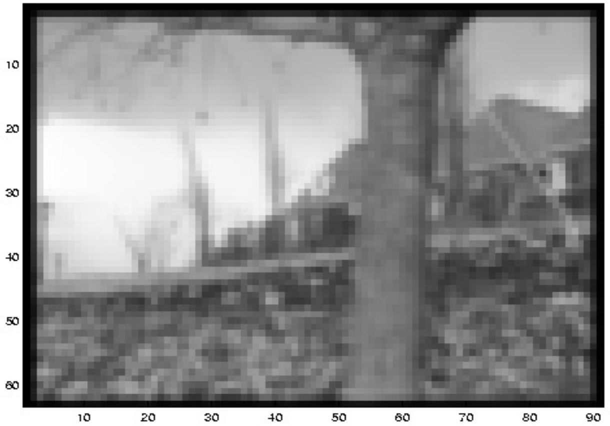
$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{\text{2nd moment matrix for feature patch in first image}} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{\text{displacement}} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

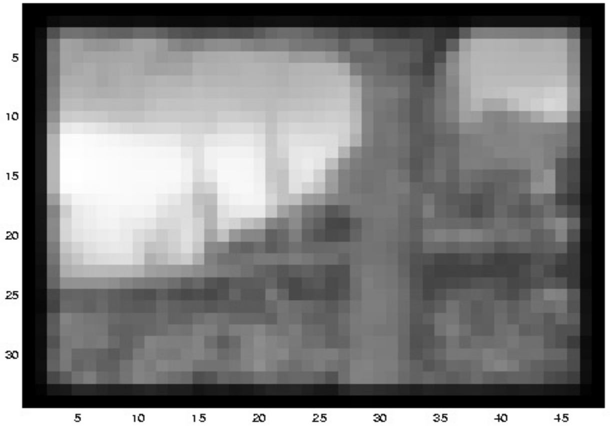
3. Shift window by (u, v): $x' = x' + u;\ y' = y' + v;$
4. Recalculate I_t
5. Repeat steps 2-4 until small change
   - Use interpolation for subpixel values

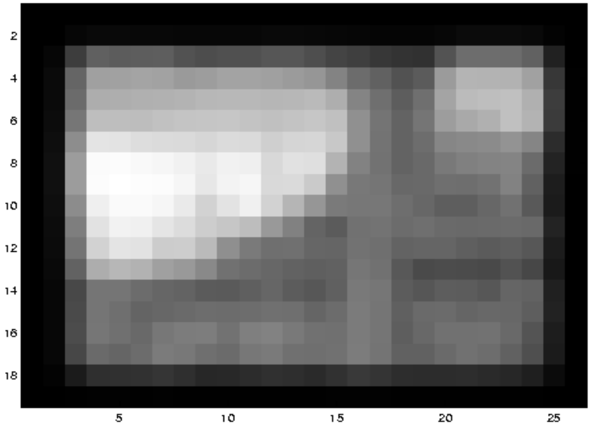
Revisiting the small motion assumption

- Is this motion small enough?
  - Probably not—it's much larger than one pixel (2^{nd} order terms dominate)
  - How might we solve this problem?

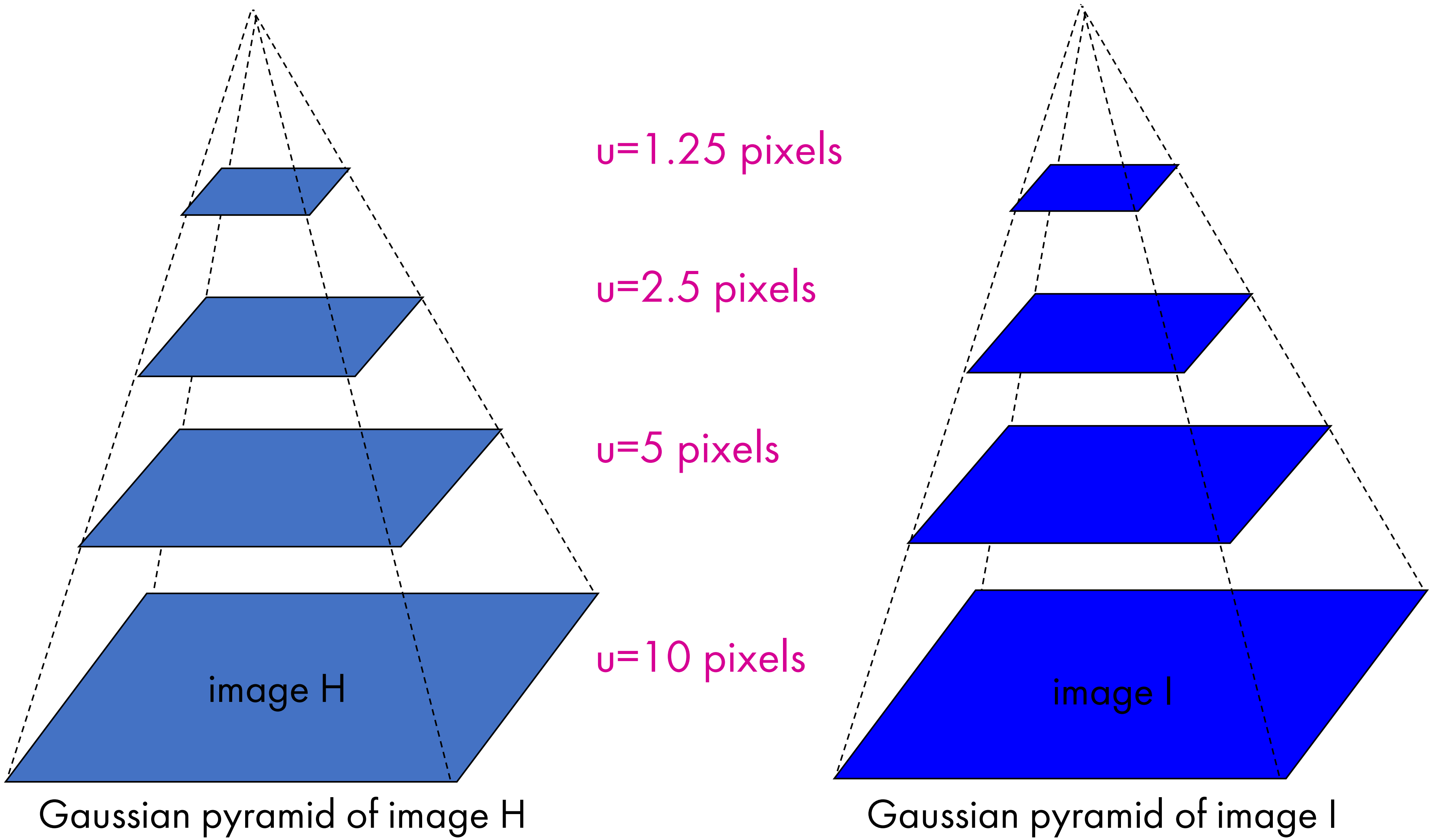
Reduce the resolution!



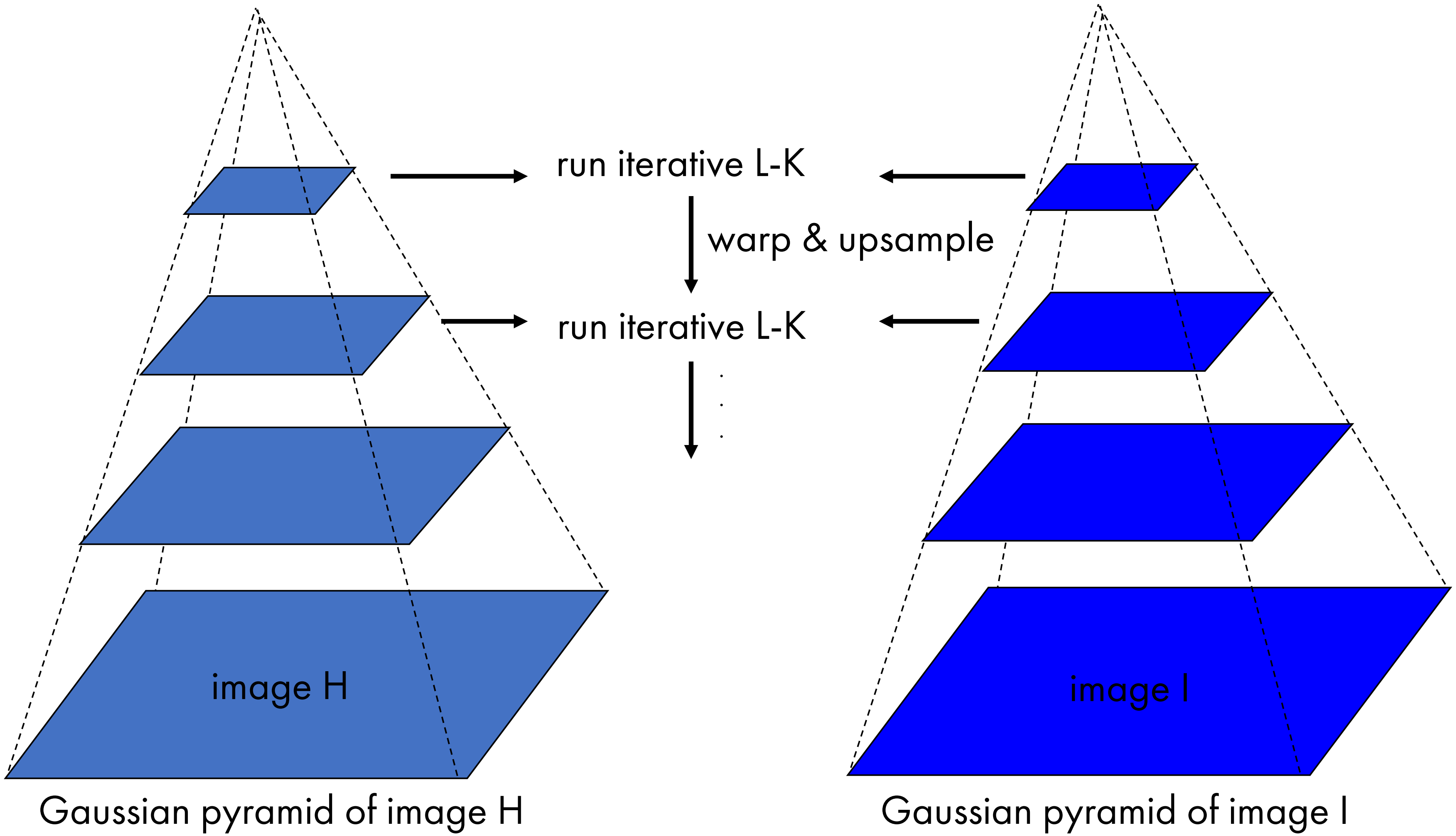




Coarse-to-fine optical flow estimation



Coarse-to-fine optical flow estimation

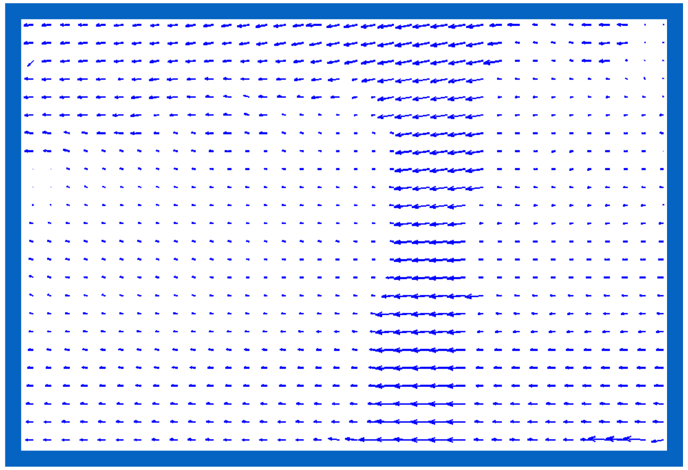


A Few Details

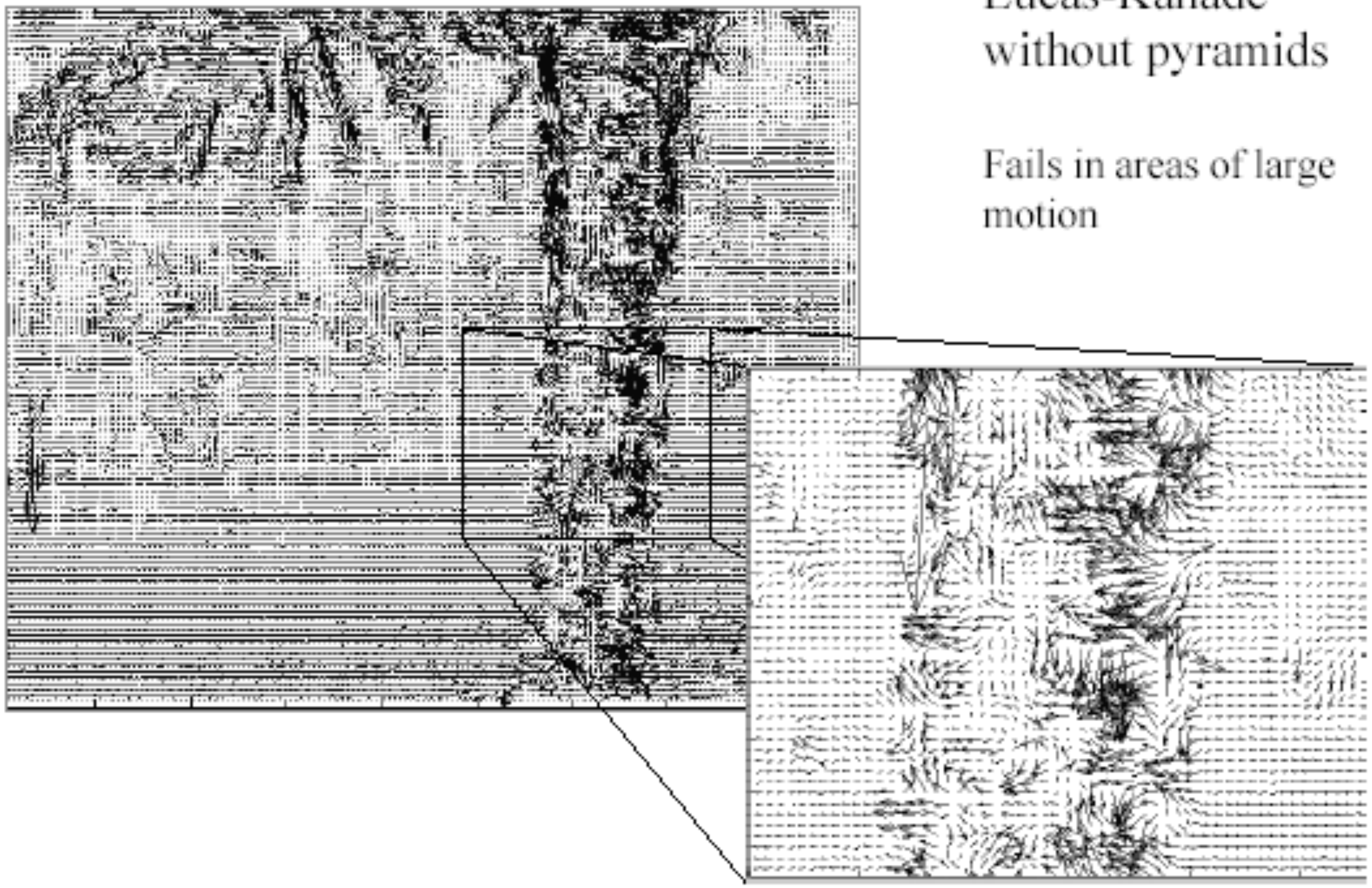
- Top Level
  - Apply L-K to get a flow field representing the flow from the first frame to the second frame.
  - Apply this flow field to warp the first frame toward the second frame.
  - Rerun L-K on the new warped image to get a flow field from it to the second frame.
  - Repeat till convergence.
- Next Level
  - Upsample the flow field to the next level as the first guess of the flow at that level.
  - Apply this flow field to warp the first frame toward the second frame.
  - Rerun L-K and warping till convergence as above.
- Etc.

The Flower Garden Video

- What should the
- optical flow be?



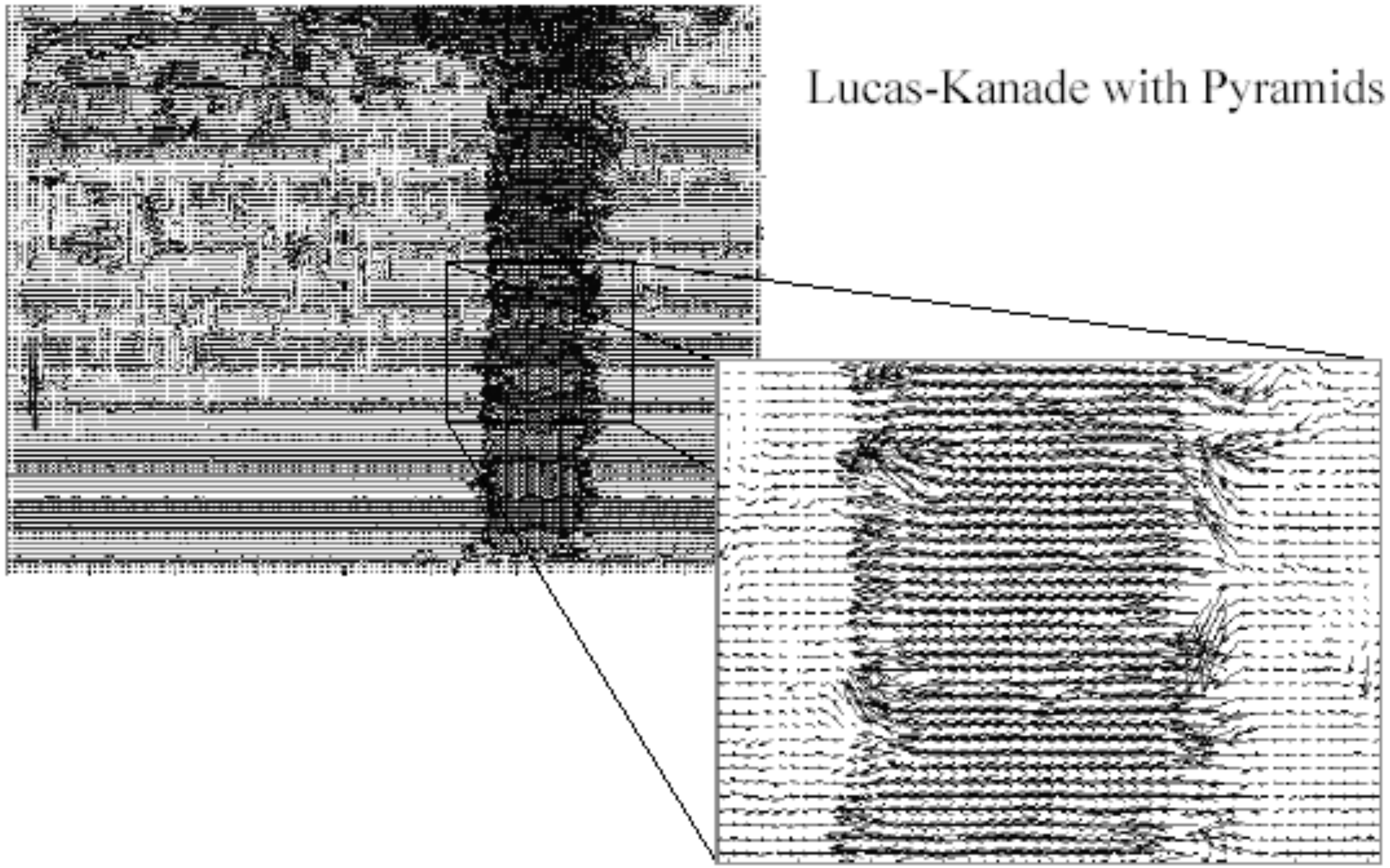
Optical Flow Results



Lucas-Kanade
without pyramids

Fails in areas of large
motion

Optical Flow Results



Next Time

- Can we also define keypoints that are shift, rotation, and scale invariant/covariant?
- What should be our description around keypoint?